17 Basic Counting Arguments

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As combinatorics explanations cannot simultaneously be clear and concise, the full explanations of these problems have been omitted, but are available on the VMT website. The authors strongly suggest that the reader closely examine the detailed solution of any problem whose solution is not immediately apparant, so that one can become familiar with the arguments needed to solve such problems.

1. How many ways can one arrange the elements of a set of size n?

   $\boxed{n!}$

2. How many ways can one choose k out of n items (order does matter)?

   $$\boxed{\frac{n!}{(n-k)!}}$$

3. How many ways can one choose k out of n items (order doesn't matter)?

   $$\boxed{\frac{n!}{k!(n-k)!}}$$

4. How many sequences $a_1, a_2, \ldots, a_k$ of positive integers exists such that $a_i < a_{i+1} \leq n$?

   $$\boxed{\binom{n}{k}}$$

5. How many ways can n distinct items be partitioned into k groups?

   $\boxed{n^k}$

6. How many ways can one travel from $(0,0)$ to (x,y) traveling only to the right and up and only between adjacent lattice points.

   $$\boxed{\binom{x+y}{x}}$$

7. How many ways can n distinct items be partitioned into k groups of sizes $s_1, s_2, \ldots, s_k$, where $s_1 + \ldots + s_k = n$?

   $$\boxed{\frac{n!}{s_1!s_2!\cdots s_k!}}$$

8. Simplfify $\binom{n}{k} + \binom{n}{k+1}$.

   $$\boxed{\binom{n+1}{k+1}}$$

9. Simplfify $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n-1} + \binom{n}{n}$.

$\boxed{2^n}$

10. How many positive integer solutions exist to the system $a_1 + a_2 + \ldots + a_k = n$?

$$\boxed{\binom{n-1}{k-1}}$$

11. How many non-negative integer solutions exist to the system $a_1 + a_2 + \ldots + a_k = n$?

$$\boxed{\binom{n+k-1}{k-1}}$$

12. How many integer solutions exist to the system $a_1 + a_2 + \ldots + a_k = n$ if $a_i > m$?

$$\boxed{\binom{n-km-1}{k-1}}$$

13. How many ways can one arrange n of one object and k of another if none of the k objects may be next to each other?

$$\boxed{\binom{n+1}{k}}$$

14. How many ways can one place k indistinct items between n other indistinct items? (Any number of the first type can go between the sucessive items of the second type).

$$\boxed{\binom{n+k-2}{k}}$$

15. Simplify $\binom{n}{0} + \binom{n+1}{1} + \ldots + \binom{n+k}{k}$.

$$\boxed{\binom{n+k+1}{k}}$$

16. How many paths of length n can be made using only left or right moves of length 1, starting on the left side of a line segment of length n?

$$\boxed{\binom{n}{\lfloor \frac{n}{2} \rfloor}}$$

17. How many ways can one place in order a total of n elements of two different types if one can at no point have placed down more elements of the first type than of the second type?

$$\boxed{\binom{n}{\lfloor \frac{n}{2} \rfloor}}$$