

9 Basic Counting Arguments

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The authors strongly suggest that the reader closely examine the detailed solution of any problem whose solution is not immediately apparent, so that one can become familiar with the arguments needed to solve such problems; such solutions to these problems and more are available on the VMT website.

1. How many sequences a_1, a_2, \dots, a_k of positive integers exists such that $a_i < a_{i+1} \leq n$?

$$\boxed{\binom{n}{k}}$$

2. How many ways can n distinct items be partitioned into k groups?

$$\boxed{n^k}$$

3. How many ways can one travel from $(0, 0)$ to (x, y) traveling only to the right and up and only between adjacent lattice points.

$$\boxed{\binom{x+y}{x}}$$

4. How many ways can n distinct items be partitioned into k groups of sizes s_1, s_2, \dots, s_k , where $s_1 + \dots + s_k = n$?

$$\boxed{\frac{n!}{s_1!s_2! \cdots s_k!}}$$

5. Simplify $\binom{n}{k} + \binom{n}{k+1}$.

$$\boxed{\binom{n+1}{k+1}}$$

6. Simplify $\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n-1} + \binom{n}{n}$.

$$\boxed{2^n}$$

7. How many positive integer solutions exist to the system $a_1 + a_2 + \dots + a_k = n$?

$$\boxed{\binom{n-1}{k-1}}$$

8. How many non-negative integer solutions exist to the system $a_1 + a_2 + \dots + a_k = n$?

$$\boxed{\binom{n+k-1}{k-1}}$$

9. How many ways can one arrange n of one object and k of another if none of the k objects may be next to each other?

$$\boxed{\binom{n+1}{k}}$$