Created Problems

Daniel L. Schafer

February 5, 2006

1 Inequalities

1.1 Simple

1. (DSchafer, 2006-02-03) Prove that for all real numbers a,b,c

$$a^2(1+b^2)+b^2(1+c^2)+c^2(1+a^2) \geq 2ab^2+2bc^2+2ca^2$$

2. (DSchafer, 2006-02-03) Prove that for all real numbers x,y,z

$$x^2+y^2+z^2 \geq \frac{9xyz}{x+y+z}$$

1.2 Standard

1. (DSchafer, 2006-02-02) Prove that for all non-zero reals a,b,c such that $a^2+b^2+c^2 \geq 1$

$$\frac{a}{bc}(1-2a^2)+\frac{b}{ac}(1-2b^2)+\frac{c}{ab}(1-2c^2) \leq a+b+c$$

2. (DSchafer, 2006-02-05) Prove that for all positive reals x,y,z where $x > y > z$

$$z^2(z-x)(z-y) \leq \frac{(x^3-z^3)(y^3-z^3)}{(x+\sqrt{xy}+y)^2}$$