REPEATING DECIMALS

By D. Tran as S. Torbert for M. Stueben (May 15, 2006 – Senior Switch Day)

Repeating (or recurring) decimals are real numbers that are expressed with an **infinitely** repeating sequence of digits:

0.111111111111... (The ellipsis denotes that the sequence continues infinitely.)

0.123123123123...

0.583333333333...

These numbers are usually expressed in a shorter and more exact fashion, with a vinculum (horizontal line) on top of the repeating sequence:

$0.\overline{123} = 0.123123123123...$

$0.58\overline{3} = 0.583333333333...$

The obvious benefit from using the vinculum is the compact notation, but it also serves the purpose of denoting the exact sequence that will repeat. For example, consider this:

$0.1234... = 0.123\overline{4}$ or $0.12\overline{34}$ or $0.1\overline{234}$ or $0.\overline{1234}$?

A repeating decimal can be converted into a fractional form. Note that this fraction **is not an approximation, but is in fact, an exact representation of the repeating decimal**. For example, many people know that $1/3 = 0.333... = 0.\overline{3}$. Using this example, simple algebra will prove this equation. Here, the ellipsis notation is used to help visualize the problem:

$$
\begin{aligned}
x &= 0.333333333... \\
10x &= 3.333333333... \\
10x - x &= 3.333333333... - 0.333333333... \\
9x &= 3.000000000... = 3 \\
9x/9 &= 3/9 \\
x &= 1/3
\end{aligned}
$$

The second step may appear to many as misleading – one could say the following:

$10x = 3.333333333...0$

But given the principles of mathematics, this is an incorrect equation. Repeating decimals are **infinitely** long (with no terminating end) and therefore will never have a number appended to the end.

REPEATING DECIMALS: CONTINUED! OR THE BIG SHEET OF $0.\overline{9} = 1$ PROOFS

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Once again, **you must remember that "point nine repeating" (PNR) is infinitely long**, and only through this qualification does it equal one. Any number of nines less than infinity, and the equation becomes invalid.

So, you've already seen one proof that "point nine repeating (PNR) equals one":

$$x = 0.999999999...$$
$$10x = 9.999999999...$$
$$9x = 9.000000000... = 9$$
$$x = 1$$

Fortunately, there are more proofs (with varying mathematical levels) that prove this known mathematical fact to be correct.

Using common knowledge as a quick proof

Using the fact that:

$$1/3 = 0.333... = 0.\overline{3}$$

you can easily show that multiplying each side by 3 will yield PNR equaling one.

The Density Property of Real Numbers

The Density Property of Real Numbers states that if two real numbers are distinct, then there must be a real number in between them. Or in terms of variables:

For any distinct real a and b, where a < b, there exists a real x that satisfies the inequality a < x < b.

In this case, there is no x that satisfies the inequality $0.\overline{9} < x < 1$. Therefore, they are not distinct – they are the same number.

Expressing PNR as an infinite sum

Numbers are basically terms added together. For example, the number 1337 would be broken down into (1 * 10^3) + (3 * 10^2) + (3 * 10^1) + (7 * 10^0). Using this method, you can break down PNR and express it as an infinite sum:

$$0.\overline{9} = (9/10) + (9/100) + ...$$
$$= \sum_{n=1}^{\infty} 9/(10^n)$$

As a sum of an infinite geometric series, we can use the following equation (where a_1 is the first term and r is the common ratio.)

$$S = a_1/(1 - r)$$
$$S = (9/10)/(1 - (1/10)) = 1$$

Expressing PNR as a limit

PNR can be expressed as:

$$0.\overline{9} = \lim_{n\to\infty} ((10^n) - 1)/(10^n)$$
$$= \lim_{n\to\infty} 1 - (1/(10^n))$$
$$= 1 - \lim_{n\to\infty} 1/(10^n)$$
$$= 1 - 0$$
$$= 1$$

In conclusion, Colton O'Connor is wrong.

RANDOM PUZZLES AND QUESTIONS

Compiled by D. Tran as S. Torbert for M. Stueben (May 15, 2006 – Senior Switch Day)

The compiled puzzles and questions below come from a number of sources, including books, the Internet, and Mr. Stueben. There are no trick questions. **Place your answers in the boxes.**

1. How many F's are in the following sentence?

FINISHED FILES ARE THE RE-
SULT OF YEARS OF SCIENTIF-
IC STUDY COMBINED WITH THE
EXPERIENCE OF MANY YEARS.

2. Punctuate the following so it makes sense:

John while James had had
had had had had had had
had had had a better
effect on the teacher.

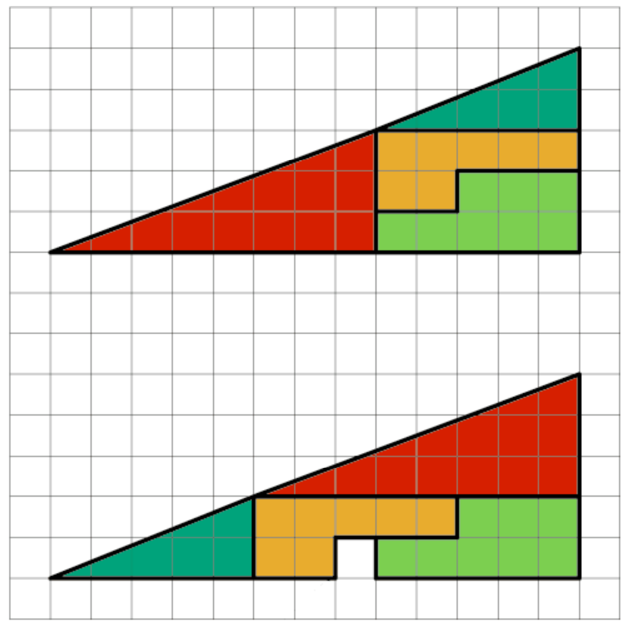
3. Give the next number in the sequence:

1
11
21
1211
111221
312211
13112221
1113213211

4. Choose the best answer:

A. All of the below.
B. None of the below.
C. All of the above.
D. One of the above.
E. None of the above.
F. None of the above.

5. Where did the missing square go?



H

(The answer is one word and starts with H.)

6. What is the name of Mr. Stueben's book?

(Do not include the subtitle.)

7. What is Mr. Stueben's license plate number?