Homework 1

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P2

a $x^5 + x = 1$. Therefore $f(x) = x^5 + x - 1 = 0$.
$f(0) = 0^5 + 0 - 1 = -1 < 0$ and $f(1) = 1^5 + 1 - 1 = 1 > 0$.
By the Intermediate Value Theoream, root $r \in [0, 1]$

b $\sin x = 6x + 5$. Therefore $f(x) = 5 + 6x - \sin x = 0$.
$f(-1) = 5 + 6 \times (-1) - \sin(-1) = -1 - \sin(-1) = \sin(1) - 1 < 0$ because $\sin(1) < 1$.
$f(0) = 5 + 6 \times 0 - \sin 0 = 5 > 0$.
By the Intermediate Value Theoream, root $r \in [-1, 0]$

c $\ln x + x^2 = 3$. Therefore $f(x) = \ln x + x^2 - 3 = 0$.
$f(1) = \ln(1) + 1^2 - 3 = 0 + 1 - 3 = -2 < 0$.
$f(2) = \ln(2) + 2^2 - 3 = \ln(2) + 1 > 0$ because $\ln(2) > 0$.
By the Intermediate Value Theoream, root $r \in [1, 2]$

P4

a Start with $a_1 = 0$ and $b_1 = 1$.

Iteration 1 $c_1 = \frac{a_1+b_1}{2} = \frac{1}{2}$. $f(a_1)f(c_1) = f(0)f(\frac{1}{2}) = (-1)((\frac{1}{2})^5 + \frac{1}{2} - 1) = \frac{15}{32} > 0$. So $a_2 = c_1 = \frac{1}{2}$ and $b_2 = b_1 = 1$.

Iteration 2 $c_2 = \frac{a_2+b_2}{2} = \frac{3}{4}$. $f(a_2)f(c_2) = f(\frac{1}{2})f(\frac{3}{4}) = (-\frac{15}{32})((\frac{3}{4})^5 + \frac{3}{4} - 1) > 0$. So $a_3 = c_2 = \frac{3}{4}$ and $b_3 = b_2 = 1$.
And so root $r \in [\frac{3}{4}, 1]$, i.e. $r = \frac{7}{8} \pm \frac{1}{8}$.

b Start with $a_1 = -1$ and $b_1 = 0$.

Iteration 1 $c_1 = \frac{a_1+b_1}{2} = -\frac{1}{2}$. $f(c_1) = 5 + 6 \times (-\frac{1}{2}) - \sin(-\frac{1}{2}) = 2 + \sin(\frac{1}{2}) > 0$ so $f(a_1)f(c_1) < 0$. Therefore let $a_2 = a_1 = -1$ and $b_2 = c_1 = -\frac{1}{2}$.

Iteration 2 $c_2 = \frac{a_2+b_2}{2} = -\frac{3}{4}$. $f(c_2) = 5 + 6 \times (-\frac{3}{4}) - \sin(-\frac{3}{4}) = \frac{1}{2} + \sin(\frac{3}{4}) > 0$ so $f(a_2)(c_2) < 0$. Therefore let $a_3 = a_2 = -1$ and $b_3 = c_2 = -\frac{3}{4}$.
And so root $r \in [-1, -\frac{3}{4}]$, i.e. $r = -\frac{7}{8} \pm \frac{1}{8}$.

c Start with $a_1 = 1$ and $b_1 = 2$.

Iteration 1 $c_1 = \frac{a_1+b_1}{2} = \frac{3}{2}$. $f(c_1) = \ln(\frac{3}{2}) + (\frac{3}{2})^2 - 3 < 0$ so $f(a_1)f(c_1) > 0$. Therefore let $a_2 = c_1 = \frac{3}{2}$ and $b_2 = b_1 = 2$.

Iteration 2 $c_2 = \frac{a_2+b_2}{2} = \frac{7}{4}$. $f(c_2) = \ln(\frac{7}{4}) + (\frac{7}{4})^2 - 3 > 0$ so $f(a_2)(c_2) < 0$. Therefore let $a_3 = a_2 = \frac{3}{2}$ and $b_3 = c_2 = \frac{7}{4}$. And so root $r \in [\frac{3}{2}, \frac{7}{4}]$, i.e. $r = \frac{13}{8} \pm \frac{1}{8}$.

P5

$x^4 = x^3 + 10$. Therefore let $f(x) = x^4 - x^3 - 10 = 0$.

a $f(2) = 16 - 8 - 10 = -2 < 0$.
$f(3) = 81 - 27 - 10 = 44 > 0$.
Therefore root $r \in [2, 3]$.

b Let ϵ_n be error after n steps.
$\epsilon_0 = \frac{1}{2}$ and $\epsilon_n = \frac{\epsilon_{n-1}}{2}$.
Therefore $\epsilon_n = \frac{\epsilon_0}{2^n} = \frac{1}{2^{n+1}}$
$\Rightarrow \epsilon_n \leq 10^{-10}$
$\Rightarrow 2^{n+1} \geq 10^{10}$
$\Rightarrow n + 1 \geq \log_2 10^{10} = 33.2$
$\Rightarrow n \geq 33$.

P6

$c_1 = \frac{-2+1}{2} = -\frac{1}{2}$
$c_2 = \frac{-\frac{1}{2}+1}{2} = \frac{1}{4}$
$c_3 = \frac{-\frac{1}{2}+\frac{1}{4}}{2} = -\frac{1}{8}$
$\ldots$
$c_n = \frac{(-1)^n}{2^n}$.
It converges to 0, but it is *not* the root. $f(x)$ is not continuous on the interval so the Intermediate Value Theorem clearly dose not apply.