Homework 11

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CP1

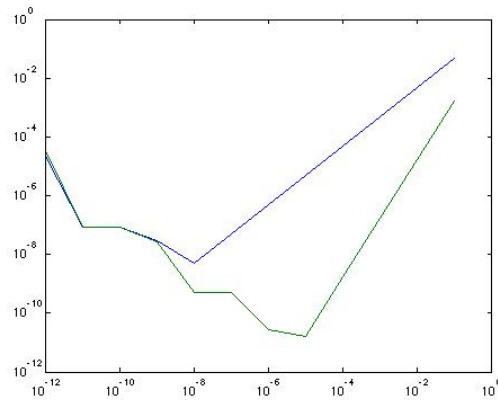
$$f(x) = \sin x - \cos x$$
$$f'(x) = \cos x + \sin x$$
$$f(0) = \cos 0 + \sin 0 = 1$$

h	$\frac{f(h)-f(-h)}{2h} - 1$
0.10000000000000	-0.00166583353172
0.01000000000000	-0.00001666658333
0.00100000000000	-0.00000016666662
0.00010000000000	-0.00000000166711
0.00001000000000	-0.00000000001565
0.00000100000000	-0.00000000002676
0.00000010000000	-0.00000000052636
0.00000001000000	-0.00000000052636
0.00000000100000	0.00000002722922
0.00000000010000	0.00000008274037
0.00000000001000	0.00000008274037
0.00000000000100	0.00003338943111

See the log-log plot, esp. at the larger values on h, it's clear that the convergence is quadratic. (the blue line is the two-point difference formula, and it's plain that it has a linear convergence.) In the error formula

$$E(h) = \frac{h^2}{6} f'''(c) + \frac{\epsilon_{mach}}{h}$$

$M = |f'''(0)| = 1$, so that the h that gives the minimum error is $(\frac{3\epsilon_{mach}}{M})^{1/3} = 6.7 \times 10^{-6} = 10^{-5}$, while the $|E(h)| \leq \frac{10^{-10}}{6} + \frac{10^{-16}}{10^{-5}} = 2.7 \times 10^{-11}$, so within the bounds & on the right order of magnitude.



P11

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \dots$$
$$f(x+3h) = f(x) + 3hf'(x) + \frac{9h^2}{2} f''(x) - \frac{27h^3}{6} f'''(x) + \dots$$
$$f(x+3h) - 9f(x-h) = -8f(x) + 12hf'(x) - \frac{18h^3}{6} f'''(x) + \dots$$
$$f'(x) = \frac{f(x+3h) - 9f(x-h) + 8f(x)}{12h}.$$

CP5

$$f(x) = \cos x,$$

$$f'(x) = -\sin x$$

$$f''(x) = \cos x$$

$$f''(0) = -1.$$

h	$\frac{f(-h)-2f(h)+f(h)}{h^2} + 1$
0.10000000000000	0.00083305560514
0.01000000000000	0.00000833330527
0.00100000000000	0.00000008334899
0.00010000000000	0.00000000607747
0.00001000000000	-0.00000008274037
0.00000100000000	-0.00008890058234
0.00000010000000	0.01190150808361

See the log-log plot, esp. at the larger values on h, it's clear that the convergence is quadratic. The minimum error appears at $h = 10^{-4}$, which is on the order of $\epsilon_{mach}^{1/4}$.