# Homework 4

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## 1.3 P7

$$W(x) = \prod_{k=1}^{20}(x-k)$$

By the product rule,

$$W'(x) = \sum_{k=1}^{20}\frac{W(x)}{x-k}$$

...which seems like it *should* be zero because $W(16) = 0$. However, one of the terms of summation, $\frac{W(x)}{x-16}$ does have a limit at $x = 16$ by cancellation. This limit, which therefore is the desired $W'(x)$ by definition, equals

$$\prod_{k=1}^{15}(16-k)\prod_{k=17}^{20}(16-k) = 15! \times (-1)(-2)(-3)(-4) = 15!4!$$

Similarly for $j \in \mathbb{Z}, 1 \leq j \leq 20$,

$$W'(j) = \prod_{k=1}^{j-1}(j-k)\prod_{k=j+1}^{20}(j-k) = (j-1)! \times (20-j)! \times (-1)^j$$

### Why is this bad?

This is bad because by having $f'(r)$ be really really big, we get a ridiculously large backward-error. The smallest difference in x from a root would produce a catastrophically non-zero $f(x)$. This is also bad because for Newton's method, $g(x) = x - \frac{f(x)}{f'(x)}$, near the roots $f(x)$ is by definition near zero, while $f'(x)$ is really big, so it breaks Newton in the sense that is will go extremely slowly.

## 1.3 CP6

With the normal $w(x)$,

$$w(x) = x^{20} - 210x^{19} + 20615x^{18} - 1256850x^{17} + 53327946x^{16} - 1672280820x^{15}$$

$$+40171771630x^{14} - 756111184500x^{13} + 11310276995381x^{12}$$
$$-135585182899530x^{11} + 1307535010540395x^{10} - 10142299865511450x^{9}$$
$$+63030812099294896x^{8} - 311333643161390640x^{7} + 1206647803780373360x^{6}$$
$$-3599979517947607200x^{5} + 8037811822645051776x^{4} - 12870931245150988800x^{3}$$
$$+13803759753640704000x^{2} - 8752948036761600000x + 2432902008176640000$$

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>> fzero(@w, 15)

ans =

  14.99454333072242
```

With the adustment to the $x^{15}$ term,

$$w(x) = x^{20} - 210x^{19} + 20615x^{18} - 1256850x^{17} + 53327946x^{16}$$
$$-1.000000000000002 \times 1672280820x^{15} + 40171771630x^{14} - 756111184500x^{13}$$
$$+11310276995381x^{12} - 135585182899530x^{11} + 1307535010540395x^{10} - 10142299865511450x^{9}$$
$$+63030812099294896x^{8} - 311333643161390640x^{7} + 1206647803780373360x^{6}$$
$$-3599979517947607200x^{5} + 8037811822645051776x^{4} - 12870931245150988800x^{3}$$
$$+13803759753640704000x^{2} - 8752948036761600000x + 2432902008176640000$$

```
>> fzero(@w1, 15)

ans =

  14.86480357685579
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By theory,

$$\epsilon = 2 \times 10^{-15}, g(x) = -1672280820x^{15} \rightarrow g(r) = -7.322815540790937 \times 10^{26}$$

$$\Delta r \approx -\frac{\epsilon g(r)}{f'(r)} = -\frac{2 \times 10^{-15} \cdot -7.322815540790937 \times 10^{26}}{14!5!} = -0.13999692354586$$

Numerically in MATLAB,

$$\Delta r = 14.86480357685579 - 14.99454333072242 = -0.12973975386663$$

Close enough, considering that $\Delta r$ is pretty large, having -1 magnitude.