Homework 4

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2.3 P15

$$A = \begin{pmatrix} 10 & 20 & 1 \\ 1 & 1.99 & 6 \\ 0 & 50 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.1 & 1 & 0 \\ 0 & -5000 & 1 \end{pmatrix} \begin{pmatrix} 10 & 20 & 1 \\ 0 & -0.01 & 5.9 \\ 0 & 0 & 29501 \end{pmatrix}$$

As we can see, the largest needed multiplier is -5000.

2.3 CP1

$$A^{(6)} = \begin{pmatrix} 2.5000 & 1.2500 & 0.8333 & 0.6250 & 0.5000 & 0.4167 \\ 1.6667 & 1.0000 & 0.7143 & 0.5556 & 0.4545 & 0.3846 \\ 1.2500 & 0.8333 & 0.6250 & 0.5000 & 0.4167 & 0.3571 \\ 1.0000 & 0.7143 & 0.5556 & 0.4545 & 0.3846 & 0.3333 \\ 0.8333 & 0.6250 & 0.5000 & 0.4167 & 0.3571 & 0.3125 \\ 0.7143 & 0.5556 & 0.4545 & 0.3846 & 0.3333 & 0.2941 \end{pmatrix},$$

$$x^{(6)} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} b^{(6)} = A^{(6)}x^{(6)} = \begin{pmatrix} 6.12500000000000 \\ 4.77566877566878 \\ 3.98214285714286 \\ 3.44233544233544 \\ 3.04464285714286 \\ 2.73645308939427 \end{pmatrix}$$

According to naive LU,

$$x_c^{(6)} = \begin{pmatrix} 1.00000000000045 \\ 0.99999999998005 \\ 1.00000000017316 \\ 0.99999999946842 \\ 1.00000000066090 \\ 0.99999999971509 \end{pmatrix}, ||x - x_c||_\infty = 6.608962266341223 \times 10^{-10}$$

Going backward,

$$b' = Ax_c = \begin{pmatrix} 6.12500000000000 \\ 4.77566877566878 \\ 3.98214285714286 \\ 3.44233544233544 \\ 3.04464285714286 \\ 2.73645308939427 \end{pmatrix}, ||b - b'||_\infty = 8.881784197001252 \times 10^{-16}$$

Relative error was $4.557630875000000 \times 10^6$ while the condition number was $3.914178445941294 \times 10^7$.

$$x_c^{(10)} = \begin{pmatrix} 0.99999999953694 \\ 1.00000008176117 \\ 0.99999724666152 \\ 1.00003470142767 \\ 0.99978801671753 \\ 1.00070821249588 \\ 0.99863891215581 \\ 1.00149967645247 \\ 0.99912054102433 \\ 1.00021262473560 \end{pmatrix}, ||x - x_c||_\infty = 0.00149967645247$$

Relative error was $8.242534426059205 \times 10^{12}$ while the condition number was $6.688889384173640 \times 10^{13}$.

2.3 P9

1-Norm

Zero

$$x = 0 \to x_i = 0 \to ||x||_1 = \sum_{i=1}^{n} |x_i| = 0.$$

$$x \neq 0 \to \exists i : x_i \neq 0 \to |x_i| > 0 \to ||x||_1 = \sum_{i=1}^{n} |x_i| > 0.$$

Multiple

$$||kx||_1 = \sum_{i=1}^{n} |kx_i| = |k| \sum_{i=1}^{n} |a_i| = |k| ||x||_1.$$

Triangle Inequality

$$||x + y||_1 = \sum_{i=1}^{n} |x_i + y_i| \leq \sum_{i=1}^{n} |x_i| + |y_i| = ||x||_1 + ||y||_1.$$

∞-Norm

Zero

$$x = 0 \rightarrow x_i = 0 \rightarrow ||x||_\infty = \max |x_i| = 0.$$

$$x \neq 0 \rightarrow \exists i : x_i \neq 0 \rightarrow |x_i| > 0 \rightarrow ||x||_\infty = \max |x_i| > 0.$$

Multiple

$$||kx||_\infty = \max |kx_i| = |k| \max |a_i| = |k| ||x||_\infty.$$

Triangle Inequality

$$||x + y||_\infty = \max |x_i + y_i| \leq \max |x_i| + |y_i| \leq \max |x_i| + \max |y_i| = ||x||_\infty + ||y||_\infty.$$