Homework 7

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November 17, 2005

3.2 P1

a

$$P_2(x) = 0 + (x-0)(0.0.63662 + (x-1.5708)(-0.40528)).$$

b

$$P_2(\frac{\pi}{4}) = \frac{3}{4}.$$

c

$$f(x) - P_2(x) = \frac{(x-0)(x-\frac{\pi}{2})(x-\pi)}{3!} f'''(c).$$

where $0 \le c \le \pi$. $|f'''(c)| \le 1$ because $f(x) = \sin x$, and therefore

$$|f(\frac{\pi}{4}) - P_2(\frac{\pi}{4})| \le \frac{(\frac{\pi}{4}-0)(\frac{\pi}{4}-\frac{\pi}{2})(\frac{\pi}{4}-\pi)}{6} = 0.24224.$$

d

$$\sin\frac{\pi}{4} - P_2(\frac{\pi}{4}) = -0.042893,$$

which is much smaller compared to the theorem's guarantee.

3.3 CP3

So the Theorem for Chebyshev says

$$|\prod_{i=1}^{n}(x-x_i)| \le \frac{(\frac{b-a}{2})^n}{2^{n-1}}.$$

The error theorem says

$$f(x) - T_n(x) = \frac{\prod_{i=1}^{n}(x-x_i)}{n!} f^{(n)}(c).$$

Because

$$\frac{d^n}{dx^n}\log x = \frac{(-1)^{n-1}(n-1)!}{x^n},$$

$$|\log(x) - T_n(x)| = \frac{1}{nx^n}|\prod_{i=1}^{n}(x - x_i)| \leq \frac{(\frac{b-a}{2})^n}{2^{n-1}nx^n} \leq \frac{(\frac{b-a}{2})^n}{2^{n-1}n} = \frac{2}{n}(\frac{e-1}{4})^n.$$

Therefore,

$$|\log(x) - T_n(x)| \leq 0.5 \times 10^{10} \Leftarrow \frac{2}{n}(\frac{e-1}{4})^n \leq 0.5 \times 10^{10} \Leftarrow n \geq 25.$$

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>> norm(logchev(25,exp(-4:0.05:4))-log(exp(-4:0.05:4)), inf)

ans =

   8.8818e-16
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My program does better than the theorem's guarantee.