

Homework 9

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CP2

Let year 2000 correspond to $x = 0$.

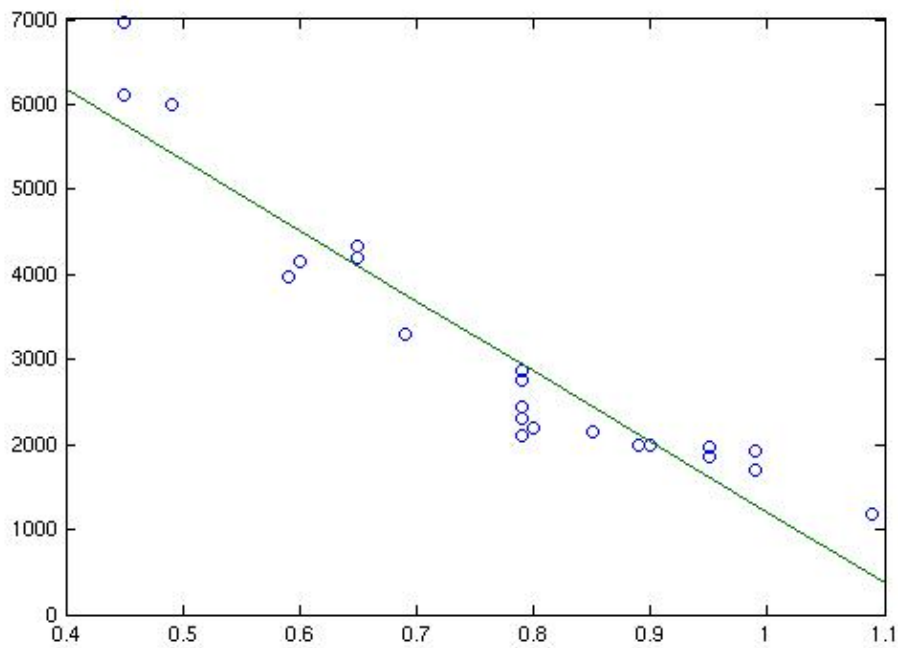
Linear $f(x) = 73.69 + 0.9693x$, $f(10) = 83.38$.

Parabolic $f(x) = 74.12 + 0.7523x - 0.07235x^2$, $f(10) = 74.41$.

Cubic $f(x) = 73.78 + 0.6084x + 0.009607x^2 + 0.01821x^3$, $f(10) = 99.03$.

CP5

$$f(x) = 9510 - 8314x.$$



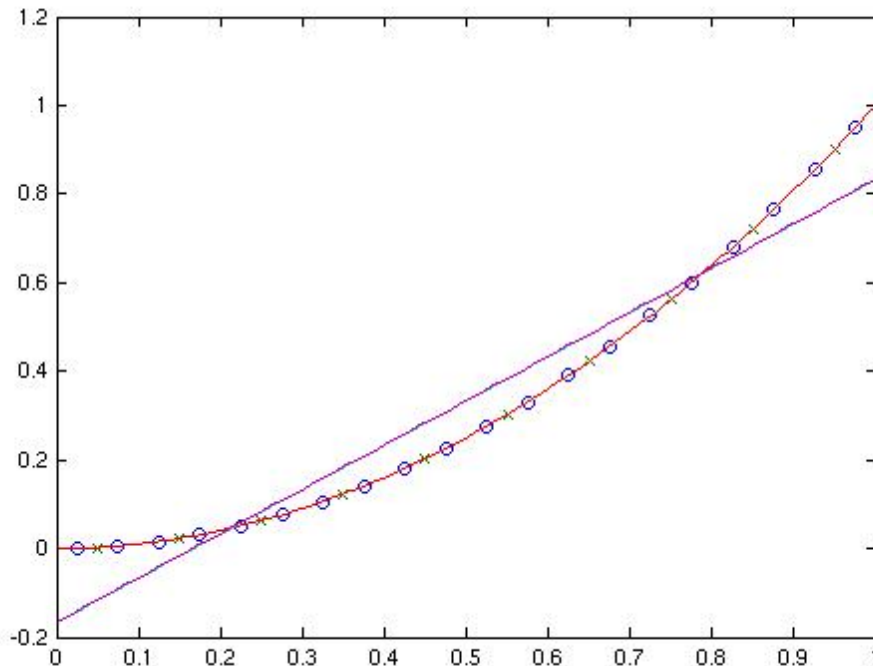
The RMS error is 2431.

Then the profit is $(9510 - 8314x)(x - 0.23)$, which is maximized at price \$0.69.

CP6

For $x = 0.05 : 0.1 : 0.95$, $f(x) = -0.1675 + x$.

For $x = 0.025 : 0.05 : 0.975$, $f(x) = -0.1669 + x$.



I rather expect that $c_1 \rightarrow -\frac{1}{6}$ and $c_2 \rightarrow 1$.

$$\begin{aligned}
 F(a, b) &= \int_0^1 (x^2 - ax - b)^2 dx = \int_0^1 x^4 - 2ax^3 + (a^2 - 2b)x^2 + 2abx + b^2 dx \\
 &= \left[\frac{1}{5}x^5 - \frac{1}{2}ax^4 + \frac{1}{3}(a^2 - 2b)x^3 + abx^2 + b^2x \right]_0^1 = \frac{1}{5} - \frac{1}{2}a + \frac{1}{3}a^2 - \frac{2}{3}b + ab + b^2. \\
 \nabla F &= \left(-\frac{1}{2} + \frac{2}{3}a + b, -\frac{2}{3} + a + 2b \right) = 0 \\
 \Rightarrow \begin{pmatrix} 4 & 6 \\ 6 & 12 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow a = 1, b = -\frac{1}{6}.
 \end{aligned}$$

The integral is the sum of all the squares of the errors of trying to approximate $y = x^2$ by $y = ax + b$ (multiplied by dx), because it's just the Riemann sum of the squares of the errors, multiplied by δx . Therefore if we minimize $F(a, b)$, then it is like finding the appropriate a and b such that the RMS error is minimized, i.e. we have the least-squares linear regression. So, Ignoring all issues of convergence, limits, and throwing all rigor out the window, a and b would be the limits of c_1 and c_2 as $n \rightarrow \infty$. So the slope of $y = x^2$ on $[0, 1]$ is 1.