A Logarithmic Randomly Accessible Data Structure

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Abstract

Making a data structure that performs like a dynamic array but functions in logarithmic time for all operations is the goal, but is by no means a trivial one. The obvious solution is to use a tree of some sort, but how?

By using a binary tree in which values are stored at the leaf nodes and each node keeps track of how many leaves there are below it, we can quickly achieve logarithmic random access, insertion and deletion in the average case but all operations are linear on the worst case. To allow balance the tree, a balanced search tree such as a Splay Tree or Red Black Tree is used. These trees are always balanced and by using their balancing mechanisms and changing them to use the random access system described above, the goal is reached.


```
public E get (int index)
{
	if (getLeft() != null)
	{
		if (getLeft().getNumBelow() >= index)
			return (E)getLeft().get(index);
		index -= getLeft().getNumBelow()+1;
	}
	if (index == 0)
	{
		splay();
		return getValue();
	}
	return (E)getRight().get(index-1);
}
```

Procedures and Methods

The primary work in devising an algorithm is conceptual, however putting it into code greatly helps in fleshing out the ideas as well as identifying flaws in the process. Similarly, when put into code the efficiency of the general case can be tested empirically.

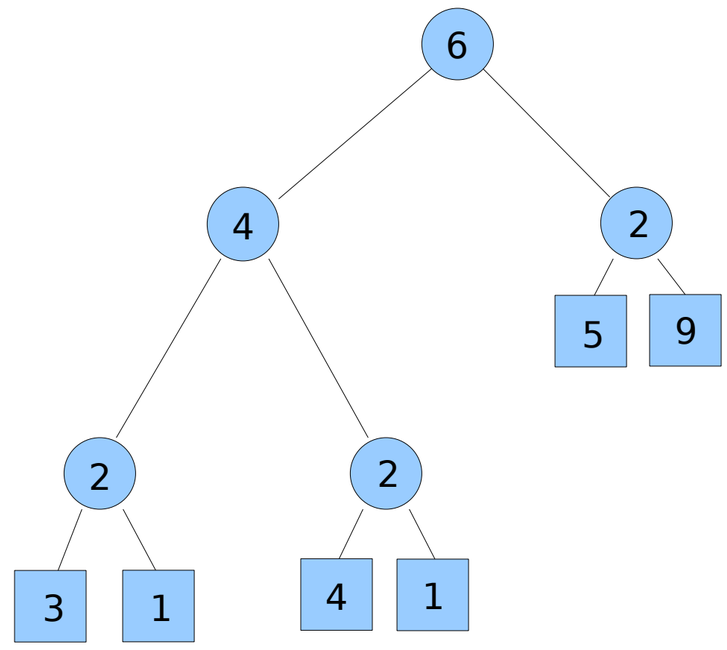
The various attempts at randomly accessible trees are implemented as classes in Java, and a separate Java program is used for testing the classes. The testing program allows for both randomly inserting and deleting data as well as sequential insertion and deletion so both random and extreme cases are testable.

The original procedure was to make a randomly accessible tree and then balance it but the focus was later shifted to modifying an already balanced tree to allow for random access.

Background

Dynamic arrays (such as vectors and array lists) are commonly used among programs to fill the places of arrays when the size is unknown and intermediate insertions and deletions are necessary. They are generally implemented as arrays whose data is shifted around when necessary and copied to a larger array when extra space is needed. Tradition dynamic arrays have O(1) random access, O(N) insertion (except to the end where it is amortized O(1)) and O(N) deletion (except from the end where it is amortized O(1)).

Red-Black Trees have been extended to allow for logarithmic access of the nth element, however although this is similar to random access it is not identical. Random access uses sequential integral keys and will change the keys of the other elements as elements are added or removed.



Results and Conclusions

I have successfully coded up both a standard Randomly Accessible Tree (as detailed above) and a Splay Tree modified for random access capabilities. Various algorithms have been designed to improve the balance and efficiency of the standard form but none can guarantee a logarithmic bound.

The Splay Trees employ a “splay” function that rotates accessed nodes to the root, thus giving them amortized logarithmic run time for all operations. I preserved this balancing mechanism's usefulness while modifying it to fit a Randomly Accessible Tree rather than a Search Tree. I have also worked to achieve similar results for a Red Black Tree.