Development of a Fractal Dimension Calculator

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Abstract

Fractal dimension is used as an index of complexity in many research applications. Currently, researchers calculate fractal dimension from raster graphics. This project introduces an alternative method by calculating fractal dimension from vector graphics. In addition, a display screen will show the user all of the steps in the calculation.
Keywords: fractal dimension, box dimension, vector graphics, vector images

1 Introduction

The term "fractal" was first coined by Benoit Mandelbrot. Used to describe geometric figures that exhibit self-similarity, fractals are now used in myriad applications. Every fractal has a numeric fractal dimension that can be used to indicate how complex the fractal is. Multiple methods for calculating fractal dimension have been formulated. The box dimension method is most commonly used in research applications.
Raster graphics are images that assign numeric values to every pixel. The values determine the color of each pixel. Vector graphics contain object types like paths and shapes to show images.
In research applications, box dimension is calculated from raster image. This paper describes how to calculate box dimension from vector images. It also describes how to display the step-by-step calculations so that users can understand every step of the process.

2 Fractal Dimension

To calculate fractal dimension using the box dimension method, a square grid of size s is superimposed over the image. N(s), the number of grids that contain part of the image, is counted. Box dimension, D, can be calculated using the following formula:

$$D = \frac{\log N(s)}{\log \frac{1}{s}} \tag{1}$$

3 Vector Graphics

This project uses Scalable Vector Graphics (SVG), a standard vector graphics language and format. Based on XML, it stores paths and shapes instead of storing a value for every pixel of an image.

4 Processing

This project also uses the Processing Development Environment. Processing, a language based on Java, comes with libraries and methods that are useful for creating graphical displays. The Candy library allows users to import SVG files, embed them in graphical displays, and search through them to find information.

5 Previous Research

Fractal dimension is used in many research applications. Previous projects have focused on the correlation between fractals dimension and aspects of the natural world. For example, Corbitt and Garbary found a correlation between fractal dimension and brown algae development. They took photographs of brown algae in different stages of development, calculated the box dimension of the photographs. (Photographs are raster images.) They knew that as the algae developed and grew older, the algae became more complex in shape. Corbitt and Garbary concluded that as algae became more complex, the fractal dimension of the algae increased.

6 Fractal Dimension Calculator

6.1 Requirements

This fractal dimension calculator must use SVG file inputs. In order to be successful, it must calculate the box dimension of the SVG imput. It must also show a display screen and let users see the calculations.

6.2 Design

Using the SVG language, images are created. These images include fractals and non-fractal shapes.
In the Processing environment, the SVG file is uploaded. Using the get() method, the coordinates and attributes of the SVG file's objects are imported into an array. The SVG image is displayed on the screen.
Now, the grid is superimposed. Starting with a large grid size, grid boxes are shown on the screen. Using a for-loop, the program goes through the object array and tallies how many grid boxes cover objects. Then, smaller grid sizes are used. A hash table keeps track of data: grid size, s, is stored in the keys, and number of grids, N(s), is stored in the values.
When the user indicates that he or she would like to stop seeing calculations, the value for D, box dimension, will be calculated using formula (1) and displayed.

6.3 Testing and Analysis

Testing the fractal dimension calculator will involve using different inputs. Many fractals have fractal dimensions that can be calculated by other methods, so those fractals will be programmed in SVG and then inputted into the Processing program. The box dimension output will be compared to the actual fractal dimension, and the error will be calculated.
Also, simple SVG files for lines and squares will be inputted to verify that the box dimension outputs are D=1 and D=2, respectively, because lines are 1-dimensional and squares are 2-dimensional.

7 Results and Discussion

Using the path and transformation attributes of the SVG language, creating fractals in SVG format has been achieved.

8 Recommendations

A useful extension of this project would be to create a function that uploads multiple SVG files at once and writes the corresponding box dimensions to an external file. This would be useful for researchers who need to find fractal dimension en masse.
Another extension would be to create a function that calculates box dimension from raster images. The Processing program would time how long the raster image calculations would take, and then compare that with how long the SVG calculations would take.

9 Appendices

9.1 Appendix A: Code

st1.svg to make a Sierpinksi Triangle

```
<?xml version="1.0" standalone="no"?>
<!--Kelly Ran  January 2008.-->
<!-- Sierpinski Triangle (aka Sierpinski Gasket)-->
<svg width="380px" height="360px" version="1.1" preserveAspectRatio="none"
  xmlns="http://www.w3.org/2000/svg" xmlns:xlink="http://www.w3.org/1999/xlink">
 <defs>
  <path id="t" d="M 0 1 L .8660254 -.5 -.8660254 -.5 Z" fill="black" />
  <g id="i1">
   <use xlink:href="#t" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#t" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#t" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
  <g id="i2">
   <use xlink:href="#i1" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#i1" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#i1" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
  <g id="i3">
   <use xlink:href="#i2" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#i2" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#i2" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
  <g id="i4">
   <use xlink:href="#i3" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#i3" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#i3" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
  <g id="i5">
   <use xlink:href="#i4" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#i4" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#i4" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
  <g id="i6">
   <use xlink:href="#i5" transform="matrix(.5 0 0 .5 0 .5)" />
   <use xlink:href="#i5" transform="matrix(.5 0 0 .5 .43301270 -.25)" />
   <use xlink:href="#i5" transform="matrix(.5 0 0 .5 -.43301270 -.25)" />
  </g>
 </defs>
 <g transform="matrix(200 0 0 -200 200 220)">
  <use xlink:href="#i6" />
 </g>
</svg>
```

sc1.svg to make a Sierpinski Carpet

```
<?xml version="1.0" standalone="no"?>
<!--Kelly Ran  January 2008.-->
<!-- Sierpinski Carpet-->
<svg width="177.4px" height="153.6px" version="1.1" preserveAspectRatio="none"
  xmlns="http://www.w3.org/2000/svg" xmlns:xlink="http://www.w3.org/1999/xlink">
 <defs>
  <path id="c" d="M 0 0 L 1 0 1 -1 0 -1 Z" fill="black" />
  <g id="i1">
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 0 0)" />
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 0 .33333333)" />
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 0 .66666666)" />
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 .33333333 0)" />
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 .33333333 .66666666)"
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 .66666666 0)" />
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 .66666666 .33333333)"
   <use xlink:href="#c" transform="matrix(.33333333 0 0 .33333333 .66666666 .66666666)"
  </g>
  <g id="i2">
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 0 0)" />
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 0 .33333333)" />
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 0 .66666666)" />
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 .33333333 0)" />
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 .33333333 .66666666)"
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 .66666666 0)" />
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 .66666666 .33333333)"
   <use xlink:href="#i1" transform="matrix(.33333333 0 0 .33333333 .66666666 .66666666)"
  </g>
  <g id="i3">
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 0 0)" />
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 0 .33333333)" />
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 0 .66666666)" />
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 .33333333 0)" />
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 .33333333 .66666666)"
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 .66666666 0)" />
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 .66666666 .33333333)"
   <use xlink:href="#i2" transform="matrix(.33333333 0 0 .33333333 .66666666 .66666666)"
  </g>
 </defs>
 <g transform="matrix(102.4 0 0 -102.4 100 150)">
  <use xlink:href="#i3" />
 </g>
</svg>
```

htree.svg to make an H-Tree

```
<?xml version="1.0" standalone="no"?>
<!--Kelly Ran  January 2008.-->
<!-- H-Tree-->
<svg width="500px" height="500px" version="1.1" preserveAspectRatio="none"
  xmlns="http://www.w3.org/2000/svg" xmlns:xlink="http://www.w3.org/1999/xlink">
 <defs>
  <path id="h" d="M 0 0.01 L 1 0.01 1 -0.01 0 -0.01 Z" fill="black" />
  <g id="i1">
   <use xlink:href="#h" transform="matrix(.5 0 0 1 0 -.5), rotate(90)" />
   <use xlink:href="#h" transform="matrix(.5 0 0 1 1 -.5), rotate(90)" />
  </g>
  <g id="i2">
   <use xlink:href="#i1" transform="matrix(.5 0 0 1 0 -.5), rotate(90)" />
   <use xlink:href="#i1" transform="matrix(.5 0 0 1 1 -.5), rotate(90)" />
  </g>
  <g id="i3">
   <use xlink:href="#i2" transform="matrix(.5 0 0 1 0 -.5), rotate(90)" />
   <use xlink:href="#i2" transform="matrix(.5 0 0 1 1 -.5), rotate(90)" />
  </g>
  <g id="i4">
   <use xlink:href="#i3" transform="matrix(.5 0 0 1 0 -.5), rotate(90)" />
   <use xlink:href="#i3" transform="matrix(.5 0 0 1 1 -.5), rotate(90)" />
  </g>
  <g id="i5">
   <use xlink:href="#i4" transform="matrix(.5 0 0 1 0 -.5), rotate(90)" />
   <use xlink:href="#i4" transform="matrix(.5 0 0 1 1 -.5), rotate(90)" />
  </g>
 </defs>
 <g transform="matrix(102.4 0 0 -102.4 200 200)">
  <use xlink:href="#h" />
  <use xlink:href="#i1" />
  <use xlink:href="#i2" />
  <use xlink:href="#i3" />
  <use xlink:href="#i4" />
  <use xlink:href="#i5" />
 </g>
</svg>
```

9.2 Appendix B: SVG Fractals Generated by Code

Figure 1: Sierpinski Triangle, 3 iterations.

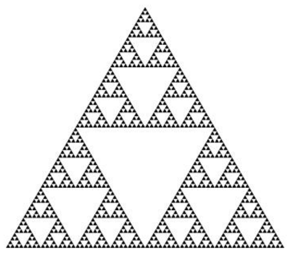

Figure 2: Sierpinski Triangle, 6 iterations.

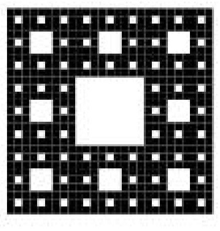

Figure 3: Sierpinski Carpet, 3 iterations.

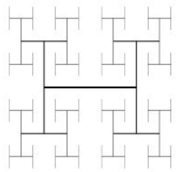

Figure 4: H-Tree, 5 iterations.

10 Literature Cited

References

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