TJHSST Computer Systems Lab Senior Research Project Fractal Dimension of Paths 2006-2007

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November 1, 2007

Abstract

This project aims to determine if fractal dimension can be used in analyzing paths between two points on a coordinate plane.

Keywords: fractal dimension, box-counting algorithm

1 Introduction - Elaboration on the problem statement, purpose, and project scope

1.1 Scope of Study

The goal is to explore the use of fractal dimension and to find how it might (or might not) be used in a nontraditional application. The results will show if a correlation exists between the fractal dimension the length of paths between any two points on a coordinate plane. This will be accomplished by generating all paths between points of a data set, calculating the fractal dimension of each path, and analyzing the results.

1.2 Expected results

If the results show that fractal dimension is not correlated to path length, it will confirm the hypothesizations of researchers that fractal dimension

should be used to analyze fractal or near-fractal objects only. This means that fractal dimension has no useful applications in non-fractal objects.

If the results show a correlation, then fractal dimension methods might be useful to analyze non-fractal objects. In fractal objects, fractal dimension is sometimes used as an index of complexity - the higher the fractal dimension, the higher the complexity (or irregularity). Perhaps fractal dimension can be used in the same way for non-fractals.

1.3 Type of research

This project is categorized as pure basic research, because the goal is to understand fractal dimension.

2 Background and review of current literature and research

Researchers have used fractal dimension to demonstrate the fractality of plants, coastlines, and other objects. Other studies have focused on the relationship between fractal dimension and complexity. For example, in 1995, Corbit and Garbary showed that fractal dimension of some species of algae is directly related to algae development and complexity (as algae develop, they grow to be more complex). Many researchers test the correlation between fractal dimension and some property of an object. For example, a study was done on fractal dimension of fractures in metal vs. tested strength of metal.

3 Procedures and Methodology

First, a set of coordinate points will be acquired. Coordinates of major cities in Northern America will most likely be used. An image-generation program, coded in Processing, will take these coordinates and create images of every possible path between any 2 selected cities. Then, a method to calculate fractal dimension must be implemented in C. The method most used in computer science is the Minkowski-Bouligand dimension, also known as the box-counting method.

I can verify the accuracy of the box-counting program by testing it on fractals with known fractal dimensions. (The fractal dimension of fractals

such as the Sierpinski Gasket can be calculated analytically.)

4 Expected Results

Graphs will be used to display results. The expected format is path length in the x-axis and fractal dimension in the y-axis. A regression analysis would be used to determine the extent of any correlation.

Future researchers could apply this project's techniques to other scenarios, either non-fractal or fractal. They could use the box-counting method to analyze natural near-fractals, like ferns or plant roots, or to analyze non-fractals like city skylines.