Applications of Stochastic Processes in Asset Price Modeling

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Abstract

Stock market forecasting and asset price modeling have recently become important areas in the financial world today. The increasing complexity of the stock market and the lucrative field of investment management has fueled breakthrough developments in mathematical stock price modeling. New financial instruments that rely on an underlying asset's price in the future to determine their current price require accurate methods of stock modeling. One method of mathematical modeling uses random or pseudorandom methods known as stochastic processes to determine an asset's price in the future. This project aims to demonstrate the flexibility and accuracy of these stochastic models by implementing them in code and testing them against empirical data.

Keywords: Stochastic processes, Brownian Motion, Financial Derivatives, Asset Price Modeling

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1 Introduction

1.1 Scope of Study

This project examines stochastic processes to predict stock price movements. Given the current price, volatility, and expected return of an arbitrary stock, several stochastic models exist to predict changes in price. There are two main models that will be implemented and tested: Brownian Motion and Geometric Brownian Motion. A standard Brownian Motion model assumes that stock prices themselves follow a random walk process. Geometric Brownian Motion, on the other hand, assumes that stock price returns, not specifically the price, follow a stochastic process. The goal of this project is to extensively test both of these models against empirical data for a single stock (IBM) to determine accuracy. Additionally, this project seeks to develop possible variance reduction techniques that improve the validity of both models.

In order to test the models against empirical data, historical prices for a specific stock (such as IBM) will be required in the past over an arbitrary time period. Yahoo! Finance has free stock price data in the past for many large companies and data from this website will be used for this experiment.

1.2 Expected results

Data garnered in this experiment will presumably reveal whether stochastic based models are accurate in predicting complex stock price movements. Calibration testing will also potentially reveal possible improvements to the current models.

1.3 Rationale

Implementation of these stochastic models and extensive testing will lead to results that help to determine the accuracy and validity of these models when they are used to predict stock prices. Several financial firms tend to use these models to price their complex financial instruments and thus require a high degree of certainty that their models are correct to prevent risk and potential losses. Invalid models used to price these instruments can lead to potential mishaps for the entire economy; this can easily be seen in the housing market collapse and subsequent chaos on Wall Street where firms did not know the correct value of their mortgage-backed securities.

Eliminating these models and developing improvements to them can lead to greater fundamental knowledge behind human behavior and mroe accurate asset pricing methods.

1.4 Type of research

This project is primarily applied research; it seeks to test models for the specific purpose of pricing assets.

1.5 Background

There have been numerous studies on developing accurate models for finance and several books have been written solely for explaining the theory behind stochastic processes and their applications. These references describe the mathematics behind these models in great rigor but fail to demonstrate empirical data to support the accuracy of the proposed models.

2 Theory

Stochastic processes are can be represented with stochastic differential equations (SDEs) that describe changes in different quantities.

Let S be the stock price, μ the drift rate (or mean), σ the volatility (or variance) of the stock, and let dZ represent a Wiener Process.

$$dZ = \phi\sqrt{dt}$$

where ϕ is drawn from a normal distribution $N \sim (0, 1)$. The SDE for Geometric Brownian Motion is given by:

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

Here $\frac{dS}{S}$ represents the return on the stock. Multiplying by S to both sides of the equation one obtains:

$$dS = \mu S dt + \sigma S dZ$$

This shows that the stock price cannot change once $S = 0$, which is a requirement for this model to accurately represent stock prices.

3 Procedures and Methodology

This project involves extensive research in the field of computational finance. The fundamentals and mathematics behind stochastic processes have been researched in detail to guarantee accurate implementation of these models.

After completing the theory behind these models, the actual models were implmented in code. Java was chosen as the prime programming language for all phases of this project. First, an RSS based class was created to retrieve real-time stock price data for an given stock from a free financial website. This will be used in later development to predict present movements of a stock price. A main statistics class was also created to act as a simple resource for calculating the mean, variance, and standard deviation for a list of a stock's historical prices over an arbitrary time period.

The stochastic models were implemented using an iterative algorithm to approximate the continuous time forms of the theoretical models. Changes in stock price were calculated for each daily trading period for a period of one year.

The next step in the project is calculating statistics for historical IBM data and using them as inputs to the model. Currently the mean (or drift rate) of the current models is assumed to be the risk-free rate of interest: 0.5%. Once these statistics are gathered the simulated IBM price over one year can be calculated at each trading day and the overall accuracy of the model can be compared with the empirical data for IBM.

Stock prices over one year can be intuitively plotted on graphs to display the price fluctuations with a smooth curve drawn through the discrete points. Charts of key prices over a year can also be provided to demonstrate potential variations in the model from the empirical data.

The accuracy of the model with respect to empirical data can be estimated by calculating the Root Mean Squared Deviation (RMSD) of the data sets. This is a measure of the average of the squared errors between the model and the empirical data. Let S_i represent the empirical price and $\hat{S}_i$ represent the simulated price. The RMSD is then defined as:

$$RMSD = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(\hat{S}_i - S_i)^2}$$

A large value indicates large deviations between the empirical data and the model, while a value close to zero represents a good fit.

4 Expected Results

I expect that the stochastic modeling techniques will approximate a stock's change in price after running many simulated trials and fine-tuning the model. Over several runs, the model should converge to the actual stock price fluctuations. The results of the project can be shown visually through graphs. For example, historical IBM stock prices can be plotted along with the simulated run of the stock to show the accuracy of the model.

If this project is successful, it could be of use to financial companies that use investment models to determine how to hedge their portfolios against risk. Improved methods of variance reduction to improve accuracy of these models also hold value for derivative pricing. Results from this project could also be used to further develop the implemented algorithms to more accurately model stock prices.

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