Application of Stochastic Processes in Asset Price Modeling

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Abstract

Stock market forecasting and asset price modeling have recently become important areas in the financial world today. The increasing complexity of the stock market and the lucrative field of investment management has fueled breakthrough developments in mathematical stock price modeling. New financial instruments that rely on an underlying asset's price in the future to determine their current price require accurate methods of stock modeling. One method of mathematical modeling uses random or pseudorandom methods known as stochastic processes to determine an asset's price in the future. This project aims to demonstrate the flexibility and accuracy of these stochastic models by implementing them in code and testing them against empirical data.

Goal

This project examines stochastic processes to predict stock price movements. Given the current price, volatility, and expected return of an arbitrary stock, several stochastic models exist to predict changes in price. There are two main models that will be implemented and tested: Brownian Motion and Geometric Brownian Motion. A standard Brownian Motion model assumes that stock prices themselves follow a random walk process. Geometric Brownian Motion, on the other hand, assumes that stock price returns, not specifically the price, follow a stochastic process. The goal of this project is to extensively test both of these models against empirical data for a single stock (IBM) to determine accuracy. Additionally, this project seeks to develop possible variance reduction techniques that improve the validity of both models.

Stochastic Processes

Geometric Brownian Motion SDE:

Stock price returns follow random process

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

$$dZ = \phi\sqrt{dt}$$

Expectation value is based on drift rate, μ

$$\begin{aligned} E(dS) &= E(\sigma S dZ + \mu S dt) \\ &= \mu S dt \\ &\quad \text{since } E(dZ) = 0 \end{aligned}$$

Variance depends on stock volatility, σ

$$\begin{aligned} Var[dS] &= E(dS^2) - [E(dS)]^2 \\ &= E(\sigma^2 S^2 dZ^2) \\ &= \sigma^2 S^2 dt \end{aligned}$$

IBM Simulated Run