Applications of Stochastic Processes in Asset Price Modeling

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Abstract & Introduction

Stock market forecasting and asset price modeling have recently become important areas in the financial world today. The increasing complexity of the stock market and the lucrative field of investment management has fueled breakthrough developments in mathematical stock price modeling. New financial instruments that rely on an underlying asset's price in the future to determine their current price require accurate methods of stock modeling. One method of mathematical modeling uses random or pseudorandom methods known as stochastic processes to determine an asset's price in the future. This project aims to demonstrate the flexibility and accuracy of these stochastic models by implementing them in code and testing them against empirical data.

Stochastic Processes

Geometric Brownian Motion SDE:

Stock price returns follow random process

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

$$dZ = \phi\sqrt{dt}$$

Expectation value is based on drift rate, μ

$$\begin{aligned} E(dS) &= E(\sigma S dZ + \mu S dt) \\ &= \mu S dt \\ &\quad \text{since } E(dZ) = 0 \end{aligned}$$

Variance depends on stock volatility, σ

$$\begin{aligned} Var[dS] &= E(dS^2) - [E(dS)]^2 \\ &= E(\sigma^2 S^2 dZ^2) \\ &= \sigma^2 S^2 dt \end{aligned}$$

Structure

After research of the theory behind these models, the actual models were implemented in code. Java was chosen as the prime programming language for all phases of this project. A main statistics class was also created to act as a simple resource for calculating the mean, variance, and standard deviation for a list of a stock's historical prices over an arbitrary time period.

The main model class is responsible for data parsing and simulation. This class reads in historical price data and utilizes the aforementioned statistical convenience class to calculate inputs to the model. Once these are determined, the simulation process begins. Currently, a Geometric Brownian Motion model is being used, but this class can easily be adopted to other models as long as they follow the same convention. These stochastic models were implemented using a discrete iterative algorithm to approximate the continuous time forms of the theoretical models. During the simulation, price changes over the given trading period (usually 1 day) are printed out to a file formatted to easily be plotted with Gnuplot. This model supports simultaneous simulations so that several different sample paths for a stock price can be plotted on the same graph.

Results & Conclusions