

# Applications of Fourier Analysis in Image Recovery

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## Abstract

The goal of this project is to explore and implement image deblurring techniques. Many of these techniques involve some sort of an image transform, and the most commonly used one is the Fourier Transform. A point spread function, also referred to as a blur kernel, can be applied to an image in the frequency domain after it has been transformed to create a blurry image. The opposite of this can be done by applying an inverse transform to a blurry image, and after removing the point spread function from the frequency domain, a deblurred image can be obtained.



Fig. 1 Original Image

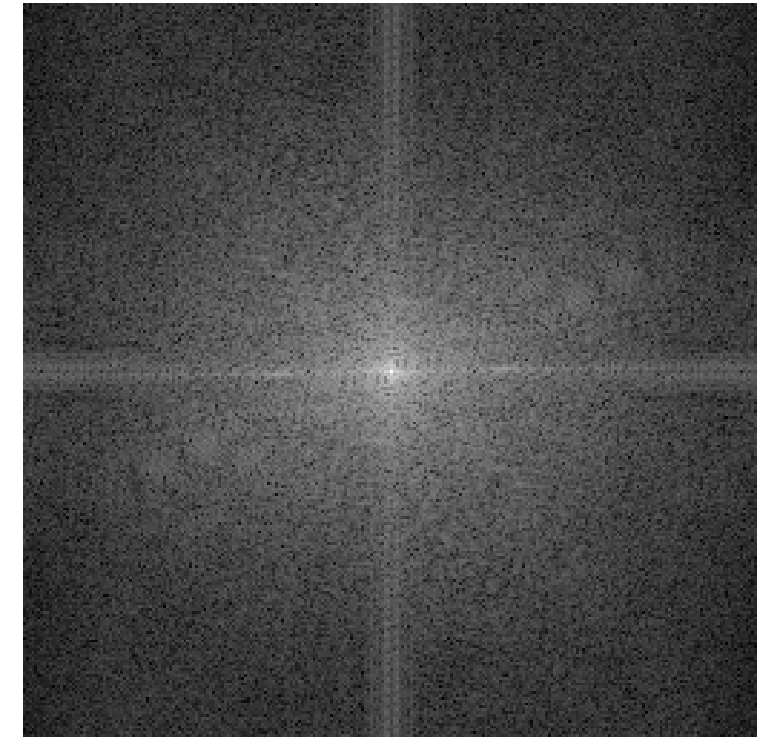


Fig. 2 Fourier Transformed Image

## Introduction

Motion blur in images is a common problem for professionals in various fields. When the image is deblurred, the usefulness of the image increases. Parts of the image that were difficult to identify can be rendered to effective clarity. This project will explore and implement image deblurring techniques. By implementing these techniques, users can efficiently remove blur from an image.

A Discrete Fourier Transform (DFT) is often too slow to be of practical use. Speed can be improved by implementing a Fast Fourier Transform (FFT). Instead of the  $N^2$  2-dimensional transforms that a DFT uses, a FFT only performs  $2N$  1-dimensional transforms. Speed improves on the order of  $N \log N$  compared to  $N^2$  for a DFT.

A Fourier Transform will transform an image in the spatial domain to the frequency domain, allowing for easier manipulation and a better view of key components in the image. By taking Fourier Transforms of a clear image and a blur filter, we can create a blurry image. The Fourier Transform of the blur filter, also known as a Point Spread Function (PSF), can be multiplied to the Fourier Transform of the original image. The actual values that are multiplied are complex and then must be inverse Fourier Transformed so that the final blurry image can be displayed. Many different types of blur can be modeled with a PSF. Therefore, if the PSF of a blurry image is known, then the image can be rather easily deblurred.

If multiplying the PSF and the Fourier Transform of the image creates a blurry image, then dividing the Fourier Transform of the blurry image and the PSF will reverse the convolution process. Knowing this, a blurry image can easily be deblurred with a non-blind deconvolution process. This process should be considered relatively simple, as the PSF is known. In cases where the PSF is not known, blind deconvolution processes must be applied, where the PSF must be estimated iteratively. Even non-blind deconvolution is not perfect and the deblurred image will contain noise that is created from the division of small Fourier values.

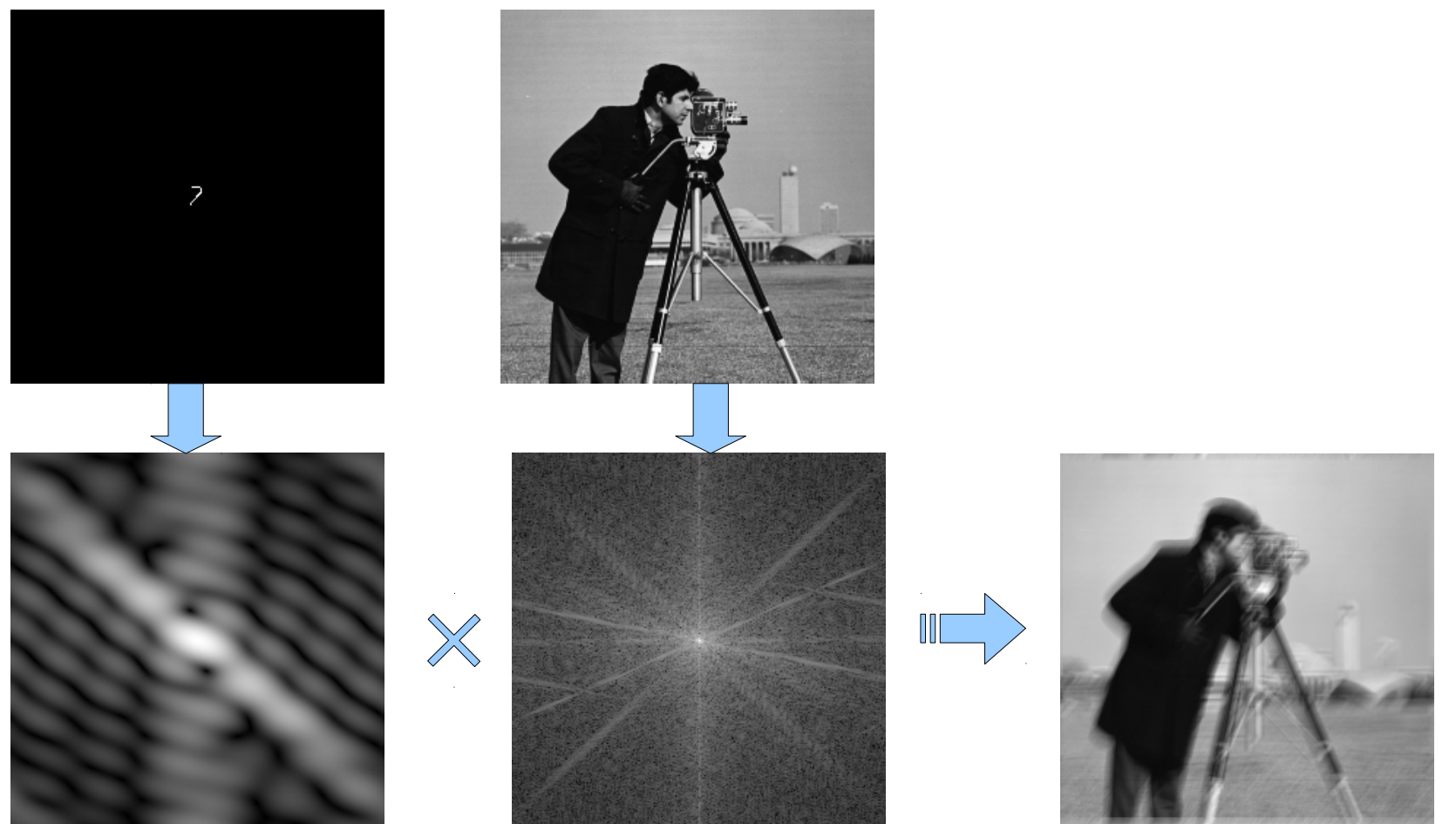


Fig. 3: Blurring an Image

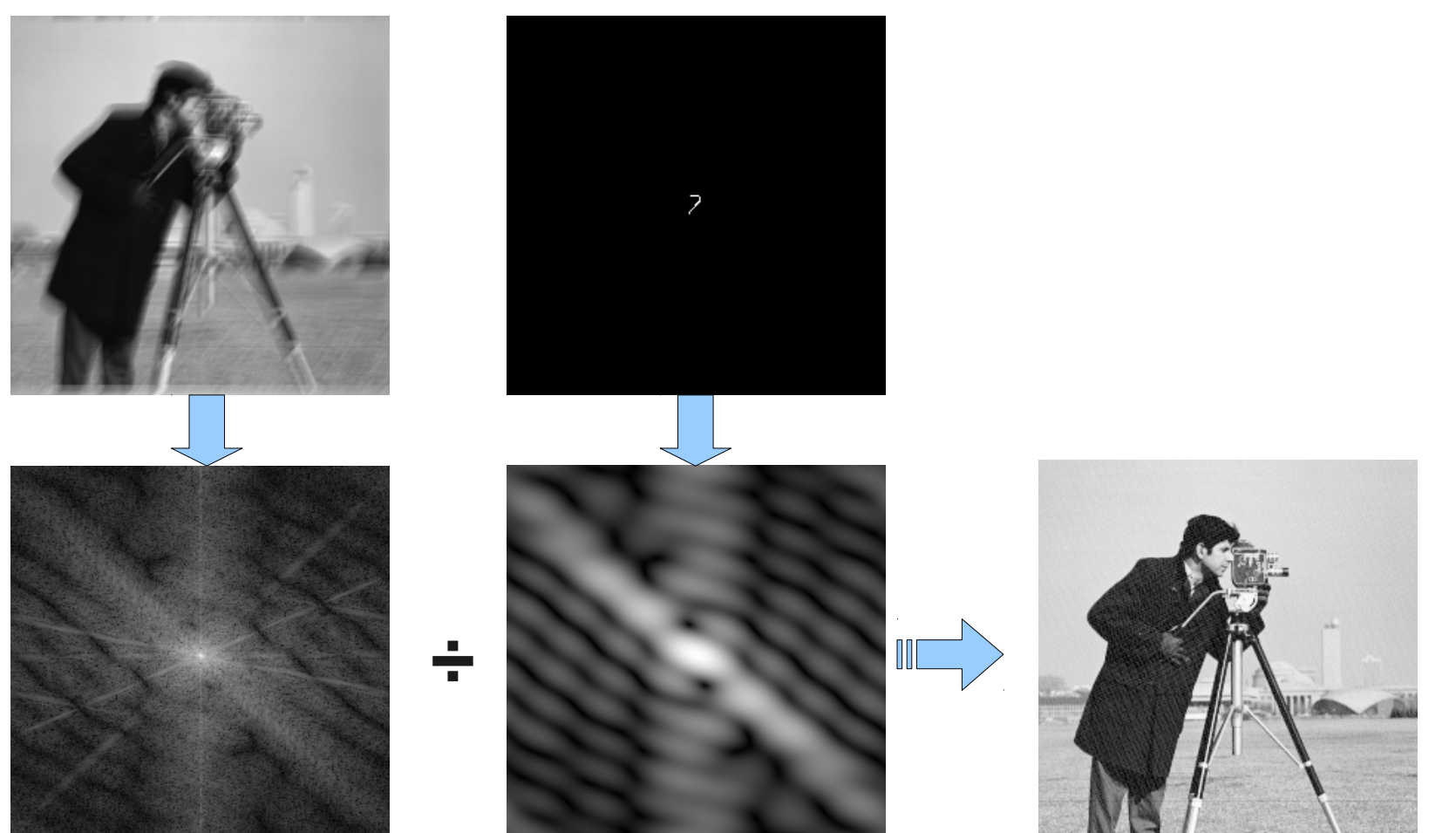


Fig. 4: Deblurring an Image