

Methods of Simulating Fluid Motion in a Shallow Context in 3-Dimensions

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April 8, 2010

Abstract

As computer graphics become more advanced and realistic, it becomes necessary to learn how to recreate real-life events in a virtual environment. The events that have proved most problematic in this regard are those that occur in nature. Water has been especially difficult because of the number of situations and environments it is found in and the vast amount of rules that govern its behavior. In this project I will investigate techniques to automate simple, shallow fluid motion found in everyday life.

Keywords: Computational Fluid Dynamics, Computer Graphics, OpenGL, Navier-Stokes, Saint Venant

1 Introduction

2 Background

Computer graphics have found many uses nowadays. It's used for animated movies, video games, and simulation software. As technology has progressed over the last few years, however, the quality of this graphic design has been increased dramatically. Photorealistic animations are now commonplace. However, it is no longer good enough to merely look realistic, they are expected to act realistic as well. Recreation of the motion of fluids has proved to be an enduring conundrum for graphic designers. This is because all computer graphics are based on the combined use of many 3-Dimensional objects. This makes modeling solids very simple, but very difficult when it comes to fluids. A deep understanding of physics is also required to recreate the motion of fluids as they are dictated by a large set of rules in nature. Because there are countless different situations and conditions that occur in nature, encompassing all of these possibilities has made it hard to code realistic fluid motion. Several different methods to approaching this problem have been researched in this field. The Navier-Stokes equations, first proposed in 1822 provide a way of understanding motion in incom-

pressible fluids. The Saint Venant equations are also used, and these are based on the Navier-Stokes equations but applied in a context that does not compensate for depth. The Navier-Stokes approach is the one that will be used in my research.

2.1 Objectives

There has been a great deal of research into the field of fluid dynamics and the rules that govern fluids. Our objective is to determine how these rules can be applied in a simulated environment. Because this is a very broad field of study, this research will focus only on liquid fluids in small, standing, and shallow contexts. This way we can isolate only a few variables to be concerned with. In doing this, we can ignore the factors of flow and currents, like those found in rivers and oceans, making the scope of the project more specified.

3 Goal

The goal of this project is to create a program that can model the motion of a small, shallow body of water when disturbed, such as ripples and waves, in three dimensions. It will allow for user input to dictate the nature and starting place of the motion, as well as for the changing of certain variables that can change how

the fluid operates. The program will allow for the view of the fluid to be changed, letting the user rotate the model to view it from any angle and zoom in and out.

3.1 Design Criteria

There will be several elements that I will incorporate into my project. The first will obviously be the window displaying the current 3 Dimensional model. I will also incorporate mouse and keyboard inputs to controls things such as the view and orientation of the model and the zooming in and out of it. In addition, several menus will be used to control key variables in the project. The program will include realistic light effects and the refraction of that light is also planned.

3.2 Procedure and Scope

In order to understand the physics behind this project, I will continue to do in depth research into what has already been learned by others about the subject and I will also discuss it with the resident physics teachers at Thomas Jefferson. I will most likely also have to do my own hands-on research on the subject of waves. I will then, from this research, discern the best way to model this motion in the form of mathematical equations and use them to manipulate the 3-D

models in my program. Because of the complexity of the general subject area, this project will only focus itself on shallow contexts and not try to recreate currents or large bodies of water.

3.3 Expected Results

The goal of this project is a very ambitious one, so while I do have high hopes for this project, I am uncertain of it's feasibility. My measure of success will be if I can have a running simulation program that accepts all appropriate inputs and shows a basic system of using these inputs in the changing of its modeling of fluids. I will most definitely need all the time I can get for working on this project to complete it. If a student next year would like to continue in the same direction as I, I would suggest the modeling of currents such as those found in rivers and oceans.

3.4 Related Work

The field of fluid dynamics is not limited to only liquids, and others have performed research regarding the flow of air and other gases [3]. This sort of project has a number of uses in areas such as aerodynamics and wind tunnel simulations. Similar methods can also be applied to the study of waves inside the earth.

4 Problem to Solve

4.1 Fluid Representation

Before even thinking about any fluid dynamics equations, we must first consider how to represent the fluid in a computer generated environment. In the case of liquids, one of the most popular options is to use a particle system, in which each particle represents one water molecule and acts independently of the others. This is the most realistic looking method. Another, similar method is to create a large 3-Dimensional grid, called a voxel field, and have each cell act as a small section of the body of water. These methods are very intensive on computer resources, as they require calculations for a very large number of different objects. These will not be used in this project, as they are far too complicated for our needs. Instead we will use what is called a height field. A height field is a 2-Dimensional matrix of points that represent points on the surface of our liquid body. This field is displayed on the XZ-plane to the viewer, and each point in the grid has a height value, which is then reflected on the Y-axis. This method is the most reasonable for our purposes, as our shallow body of water is not concerned with the flow of water below the surface.

4.2 Computational Fluid Dynamics

After deciding on how to represent liquid in 3-D space, we must then figure out how to make it move. The most common method is the use of the Navier-Stokes equations, discovered in the 19th century. These equations take into account variables such as pressure, gravity, viscosity, and density to describe velocity vectors for every point in a body of liquid. This equation is applied in many different ways depending on the context and often combined with other mathematical ideas. Because our situation only concerns a shallow body of water and thus we only care about its motion on the surface, the Navier-Stokes equation can be simplified to disregard the motion underneath the surface of the body of water. Our program will use this equation and apply it to every point in the height matrix which will in turn resolve that point's height and display it accordingly. Other methods include using the Saint-Venant equations, which are modified versions of the shallow water equations, and work in a very similar way to the Navier-Stokes equations.

4.3 Assumptions

To properly derive an equation that we can properly translate into code

for our program, there are several assumptions that we must first make. The first is that the only thing used is a height map to represent the surface of the water. This means that although we do not concern ourselves with any water underneath the surface, that also means that the motion on the surface is severely limited. It cannot splash; for that, we must use a particle system. The second assumption is that we can ignore the vertical velocity of the water points. The

third and final assumption is that any given point represents a column of water under it, and that the horizontal velocity at any point is constant through that column.

4.4 Discussion

In order to properly implement the equation into the program, after we have made our assumptions, we are left with a form of the Navier-Stokes equation as shown:

However, the simplified version of the Navier-Stokes equation has one great flaw: it does not take into account the bottom of the water versus the height. This means that it is possible for the height to be below the floor of the body of fluid, causing the depth to be negative. This also means that the equation does not conserve volume correctly. This means that I need to check that that doesn't happen manually. With a simple method, if the volume of the body doesn't stay the same at any time, the extra positive or negative volume will be distributed across the body. This way, it will always stay the same mass. I also need to, at the beginning of the loop, create the Matrix A, and use that and the 3rd equation shown to solve for all of the heights. This is the correct method, which I was not using before. Since switching to this method, I found I could totally disregard acceleration and velocity, meaning I would not need to store them in the array of every point along with the heights.

5 Results

The current results are on track with my plan. The Navier-Stokes equation has been coded before, but upon further analysis, it was not all that I needed. I needed to change my approach to include the new matrix,

use the heights of previous points, and compensate for any changes in volume. Because of this change in methodology, I could get rid of the code that stores the acceleration and velocity, so now the height field is only a 2D array and not a 3D array. When run, my program produces motion that is close to my goal, albeit on a much smaller scale (only an 11x11 height field) and only in a single direction. I will work to get the program moving in both the X and Y directions.

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