

# Statistical Analysis of Fluctuating Variables on the Stability of Predator Prey Relationships

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## Abstract

Simple predation prey simulations greatly simplify the problem by assuming multiple variables to be a constant value, and thus are not very good predictors of a natural environment. In reality, a system will have multiple possible variables such as the size of the habitat, initial population sizes of both predator and prey, reproduction rates, the probability of a predator succeeding in killing a prey, the energy gained from either consuming a prey or consuming vegetation, and much more. This two part project will first compare a simulation that considers organism behavior and intelligence with one that is simple and random. It will then statistically analyze the effects, specifically the difference in stability of the simulation, of incrementing such changes listed above in a two species system.

**Keywords:** Predator, Prey, Predation, Behavior, Intelligence, Stability, Fluctuation, Agent Based Modeling

## 1 Introduction

This project involves writing a program that models a single predator, single prey system, where both the prey and the predator depend on a food source to survive. Both prey and predator asexually reproduce offspring with random attributes. In order for a stable predator prey system to exist, a small group of prey must escape the increasing predator population. That prey

population must also reproduce slightly faster than the rate at which the predators die. This allows the predator population a chance to bounce back into survival, and the cycle continues hypothetically. With a simple predator prey system, the prey will all die off from the predators. Since all the prey are equal, none will be able to escape predation. The predators will then quickly die off from starvation. In order for a stable ecosystem to exist between predator and prey, some prey must be intelligent enough to escape predation and reproduce to continue feeding the predators. The first portion of the experiment involves demonstrating the effects of incorporating will analyze the magnitude of the effect of incorporating behavior and intelligence into a simple predator prey simulation. Having experimental evidence to support these theories would advance the validity of predator prey mathematical models.

The second part of the project involves conducting statistical analysis of the changes in stability when slowly incrementing certain variables in a predator prey system such as initial population numbers, the size of the environment, and the reproduction rates of both species. Using regression and hypothesis testing, we can determine if two variables are correlated in any way. We can then use this information to extrapolate the stability of a human controlled habitat such as a zoo or wilderness preservation after inputting these initial values. Of course, these values would only be a rough estimate of the true stability, but this project would be one step closer toward creating an artificial environment.

## 2 Background

There are a few permanent traits that most animals have. These include genetic traits (sex, size, etc.) and preferences (whether it likes brightness, darkness, cold, warmth, etc.) There are three main temporary factors to consider when an animal decides how to behave as well: hunger (should it eat?), libido (should it mate?), and fear (should it take evasive action?).

Two more complicated characteristics of the prey and predator are the Allee effect for the prey and prey choice for the predator. The Allee effect suggests that for smaller populations of prey, reproduction and chance of survival both decrease. This effect disappears as population size increases. There are two models to predict the food of predators. These assume that prey size and prey abundance are the only availability factors of importance to predators. One model suggests that the predator consumes prey as they

are encountered, and the other assumes that predators feed to maximize energy intake. These, along with other general characteristics will have to be considered when writing the program and analyzing results.

In a perfect world, all prey and predators will have a radar system and know the exact positions of all enemies. Using this knowledge, each organism could hypothetically calculate heuristics and determine the best possible behavior. However, in a world with imperfect information, each prey will have to calculate the most favorable actions with the highest probability of benefiting. For example, it is probably not favorable for an animal to reproduce when its energy level is low.

### 3 Development

here are three different actors in this program. In the GUI, the predators are represented by black dots, the prey by blue dots, and the patch actors are unit squares tiled in the area. The simulation takes place in an  $N \times N$  map with  $P$  initial predators and  $Q$  initial prey. The value of these variables may be modified before running the simulation with the GUI. The predators feed on the prey and the prey feeds on the growing vegetation in the patch, represented by green dots.

In the simple simulation, each organism is given a metabolism of 1, a random location, and an initial energy level between 5 and 10 units. In each step, both predator and prey move randomly. If a predator is less than one unit away from a prey, it will attack with a 0.5 probability of killing the prey. It will then have a probability of 0.1 of reproducing. If it succeeds in reproducing, it's total energy is divided between itself and its offspring. The prey also moves randomly, collecting the vegetation on the patch it's on if there is any. Prey also reproduce the same way predators reproduce. The simulation ends when one organism is extinct.

The simulation that considers behavior and intelligence is slightly different. During the setup, each organism is given a metabolism of 1, a random location, an initial energy level between 5 and 10 units, a random speed value centered at 1.0 with a standard deviation of 0.1, a random vision value between 2 and 5, and a random skill value centered at 0.5 with a standard deviation of 0.1. The metabolism rate is subtracted from the total energy of each organism at each step, and the organism dies from starvation once that energy reaches 0. The vision of the organism is a compilation of all of its

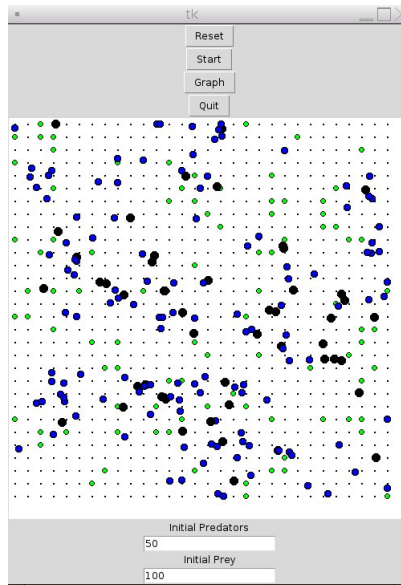


Figure 1: Demonstration of the GUI

senses used to detect prey and predators. The higher the vision, the farther a predator can see a prey and vice versa. The skill value is a measure of the organism's innate adeptness, which will be used to calculate the outcome of predator-prey encounters. The higher an organism's skill value, the higher chance it has in leaving an encounter victorious.

Each prey has a list of hunters that it is aware of. During each step, a prey that is not hungry will move a distance of  $\text{speed} * 1$  unit of time away from the average location of all its current hunters. However, if the prey is hungry, it will move to the closest patch of food it can see. The prey will then collect any food in the current patch. It has a probability of  $\text{PreyRate} * \text{NumPrey} * (1 - \text{NumPrey} / \text{PreyCapacity}) / \text{NumPrey}$  of reproducing, where in the program  $\text{PreyRate}$  ranges from 0.05 to 0.15,  $\text{NumPrey}$  is the current number of prey, and  $\text{PreyCapacity}$  is the total capacity of the environment for prey. The  $\text{PreyCapacity}$  is equal to one third of the number of patches in the simulation. If the prey succeeds in reproducing, its total energy is divided between itself and its offspring. Like predators, prey are inhibited from reproducing if their energy level is less than 4. The predator has a target prey that it chases at each step until the target is out of site or dies. During each step, a predator with a target will move forward a distance of  $\text{speed} *$

1 unit of time toward the prey, while a predator without a target will choose the closest prey within its vision radius, and move forward a distance of speed \* 1 unit of time toward the prey. If there is no prey within its vision radius, it will move randomly. If the distance between the predator and its target is less than the predator's speed \* 1 unit of time, then the predator will attack with a chance of killing it equal to  $(1 - \text{prey.skill}) * (\text{self.skill}) * \text{KillRate}$ , where the KillRate is between 0 and 1. The predator will then have a probability of  $\text{PredRate} * \text{NumPred} * (1 - \text{NumPred} / \text{PredCapacity}) / \text{NumPred}$  of reproducing, where in the program PredRate will range from 0.02 to 0.12, NumPred is the current number of predators, and PredCapacity is the total capacity of the environment for predators. The PredCapacity is equal to one sixth of the number of patches in the simulation. If the predator succeeds in reproducing, its total energy is divided between itself and its offspring. Because organisms are not likely to reproduce with low health levels, the program prevents predators with energy levels under 5 from reproducing.

## 4 Tests and Analysis

Both the simple simulation and behavioral simulation programs were ran with the same initial map size, predator population, and prey population. The graphs of population vs. time for both the simple model and the behavior model are shown in the appendix as Figure 1 and Figure 2 respectively. Multiple trials were run, but they were mostly similar in outcomes. In the simple model, the predators immediately killed off all the prey and then died of starvation. In the behavior model, there were a few select stronger prey that survived from the initial predators. However, they could not reproduce fast enough to support the dying predator population, leading to a one organism system.

In the second experiment, multiple simulations were run with different initial values, with 30 trials for each set of points. The initial predator population ranged from 50 to 100, the initial prey population ranged from 100 to 200, the environment size ranged from 400 patches to 1600 patches, the predator reproduction rate ranged from 0.02 to 0.12, the prey reproduction rate ranged from 0.05 to 0.15, the kill rate ranges from .5 to 1, the energy the predator gains from killing its target ranges from 5 to 15, and the energy a prey gains from a unit of food in a patch ranges from 3 to 6. The simulation was run with these initial values 30 times, and the stability of each trial was

recorded. The mean and standard deviations of these values were then taken and recorded. The data was then outputted into a table. For the complete data, contact me at zhengyu.lenny.li@gmail.com.

## 5 Simple vs. Behavior Results

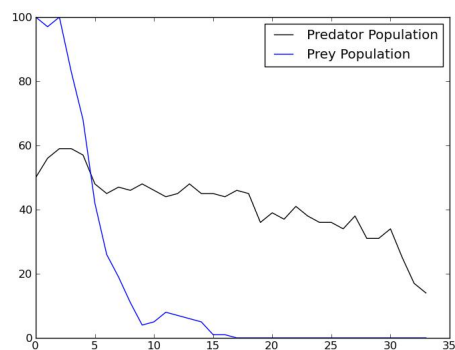


Figure 2: Population vs. Time of Simple Model

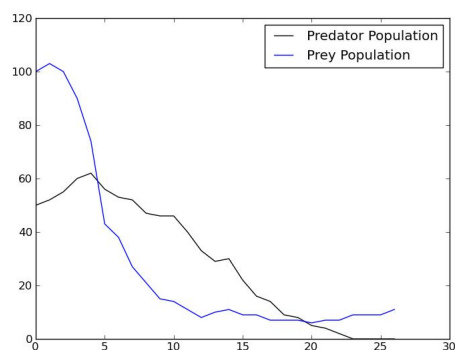


Figure 3: Population vs. Time of Behavior Model

## 6 Simple vs. Behavior Discussion

We see that the behavioral model is closer to an oscillating stable predator prey system with minimal changes to incorporate behavior. It is already one period more accurate than the simple model. With more incorporation of behavior and probably some changes to allow the prey to reproduce quicker than the predators die, a more stable system may be created.

## 7 Multiple Regression Summary

Regression Statistics	
Multiple R	0.478401111
R Square	0.228867623
Adjusted R Square	0.2287285
Standard Error	31.38794032
Observations	33264

Figure 4: Regression Statistics

	Coefficients	Standard Error	P-value
Intercept	186.4541457	2.186833616	0
Number of Predators	-0.556146663	0.011687164	0
Number of Prey	-0.117070707	0.005442214	6.0169E-102
Predator Reproduction Rate	-522.5348725	15.39290453	2.4174E-248
Prey Reproduction Rate	15.32512626	10.53880129	0.145910785
Kill Percentage	-76.17841682	1.007702145	0
Energy from Prey Gain	1.744611291	0.105388013	2.62974E-61

Figure 5: Multiple Regression Constants and P-Values

## 8 Multiple Regression Discussion

After conducting a multiple regression analysis, I found an R-squared value of .22, which means that approximately 22% of the variation in the stability of the system can be explained by the correlation between the stability and the independent parameters. This may seem low, as we are generally hoping for a value close to 1, but imagine a basketball player improving his free-throw percentage by 22%.

The p-values for each variable, except the prey reproduction rate, were all below our alpha value of 5%. This means that there is a statistically significant relationship between each independent variable and the stability of the system. The p-value is a measure of the probability that the correlation obtained is from pure chance if you assume the variables are not correlated. Since the p-values are so low (close to 0) we can conclude that there is some correlation between each of the variables and the stability of this system. The p-value of the prey reproduction rate, however, was 15%, so we cannot conclude any statistical significance. Our best fit model was found to be:  $\text{Stability} = 186.4541 - 0.5561 * \text{Initial Predator Population} - 0.1171 * \text{Initial Prey Population} - 522.5349 * \text{Predator Reproduction Rate} + 15.3251 * \text{Prey Reproduction Rate} - 76.1784 * \text{Kill Percentage} + 1.7446 * \text{Energy from Prey Gain}$ .

## 9 Conclusion and Applications

Today, Wildlife Preservations like Yellow Stone spend tons of money sampling the populations of their many protected species in the park, and then decide how many more animals to purchase to keep the human-controlled ecosystem stable. My experiment introduces a very basic model to render this entire time and money consuming process unnecessary. Since these parks are, for the most part, controlled by humans, we can model the initial conditions of the system and use multiple regression to calculate the variables that would lead to the most stable system. Of course, actual preservation areas have more than just the one predator one prey system, and more complicated programs must be written to model real-life situations. However, my project leads to the foundation of future research. Money and animal lives will be saved in the long run.



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