Abstract

Homology groups are an important concept in algebraic topology. The homology groups of a simplicial complex represent certain characteristics of the structure that can distinguish it from other simplicial complexes.
Methods to compute homology groups from a triangulation are well studied, but most such methods focus on the matrix operations required to reduce a matrix to Smith normal form. This paper investigates the use of geometric structure of Euclidean space in order to create space and time efficient algorithms to compute homology groups of complexes embedded in Euclidean space.

Introduction

Topology is the study of geometric structure on sets. One of the big questions of topology is to determine when are two spaces essentially the same. Algebraic topology is a branch of topology which computes algebraic objects which remain invariant when the underlying structure remains the same. These algebraic invariants include homology groups which encapsulate certain characteristics about the underlying structure of the space. In this paper I consider the homology groups of a certain class of spaces known as simplicial complexes. Algorithms to compute these homology groups are known, but they are generally inefficient. This project will investigate the efficiency of various algorithms to compute these homology groups. Improving the efficiency of such algorithms is applicable in experimental mathematics, where computers can perform computations beyond the reach of human work.

Previous Results

Known algorithms for computing homology groups include polynomial time reductions of a matrix to Smith normal form, an expected quadratic time algorithm for computing the homology groups of simplicial complexes with sparse boundary matrices given by Bruce Donald, and geometric methods for three dimensional manifolds in three dimensional Euclidean space given by Dey and Guha. These results will be extended to include manifolds of dimensions higher than three.

Definitions

Results

I expect to obtain an algorithm to compute the homology type of a four dimensional manifold embedded in four dimensional space. It is likely that the algorithm will also extend to higher dimensions as many topological techniques have special cases for three dimensions and below while working in general for dimensions four and above.

Discussion