An Analysis of a Dynamic Application of Black-Scholes in Option Trading

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Abstract

For decades people have invested in the stock market in with stocks, options, and bonds. Various groups of people have worked towards modeling the stock market with mathematics. One of the earliest is Black-Scholes. Developed by Fischer Black and Merton Scholes in 1973, it remains one of the most prevalent tools used by European investors today. However, the Black-Scholes model is catered toward European options, which have a definite time towards maturity. American stocks, on the other hand, do not have such constraints and can be bought and sold at any time. This project explores the ways in which Black-Scholes can be applied to the more dynamic American option trading market. The major focus of study will be comparing call and put values generated by the Black-Scholes model to historical call and put values.

Keywords: genetic algorithms, financial modeling

1 Introduction

1.1 Background

For decades people have invested in the stock market in with stocks, options, and bonds. Various groups of people have worked towards modeling the stock market with mathematics. One of the earliest is Black-Scholes. Developed by Fischer Black and Merton Scholes in 1973, it remains one of the most prevalent tools used by European investors today. The Black-Scholes model follows traditional Geo-Brownian motion. Since its development in 1973, a variety of other models have been created. In 1977, Phelim Boyle created the Monte Carlo method for option pricing for European options. His creation was later expanded by Broadie and Glasserman for option pricing in an Asian market in 1996. In 1979, Cox, Ross, and Rubenstien took a binomial tree approach to option pricing with a discrete time-frame. While most of the above methods cater to European markets, Roll, Geske, and Whaley developed an analytic solution and a formula for American call options later on. The Black-Scholes involves several main variables: stock price, strike price, volatility, time until maturity, and the risk-free interest rate. Stock price denotes the current market price of the stock, and strike price denotes the price that the option can be exercised at. Time until maturity is the time until the option can be exercised; this value is often measured in years. The risk-free interest rate is the rate of return; in this experiment, the risk-free rate will be equal to the rate of a US Treasury Bond. Finally, volatility is the measure of variation in returns of a stock option. In the American stock exchange, volatility is often expressed in terms of beta.

1.2 Purpose

The Black-Scholes model is a popular tool in helping European investors determine the calls and puts of European options. The goal however, is to use the Black-Scholes model to estimate call and put values of American stock options. Another underlying purpose is to adapt the European model into an American model. Because unlike European options, which are held until a certain time (maturity), American options can be sold at any time, making the market much more dynamic. Conforming the model to American parameters can be a helpful investment tool to traders and investors alike.

The purpose of the research project was to investigate the Black-Scholes model used in an American market. As previously stated, the American market is much more dynamic and volatile than a European model. The project will seek to answer questions regarding the validity of the Black-Scholes model. How do the call and put values generated by Black-Scholes compare to actual historical values? Do real life scenarios conform to the assumptions of the model?

1.3 Scope of Study

Most of the work revolves around the Black-Scholes model and the input of its required variables. Because the B-S model revolves around a constant time period, it will first be evaluated as such. The project will incorporate an excel model that takes the input of required values and generates call and put values from the input.

1.4 Methodology

The program will be coded in excel and then Java consisting of two main components. One is the stock class, holding all the required inputs. The other, perhaps the most important part of the project, is the Black-Scholes class containing the B-S formula and model. Outputs will be a series of numerical data. This data will then be outputted into a spreadsheet and graphed as a time-series plot. Inputs will be price, volatility, and interest rates. For testing purposes, pre-determined values will be used. However, historical data on stocks and options can also be used as inputs. To determine accuracy, the price can be compared to a calculator or historical data. The computation of each variable in the output will be a different sub-function, and each will be debugged and checked separately.

2 Project development

Black-Scholes calculates the value of call options using two differential equations.

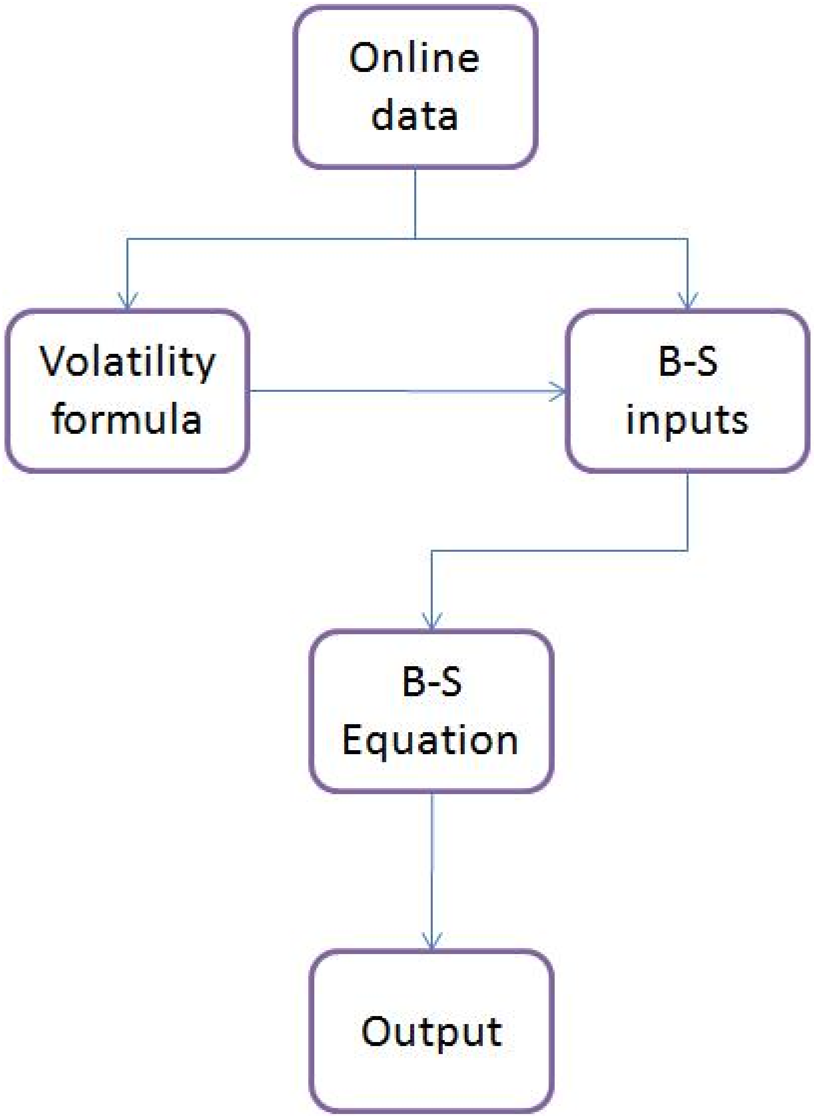
The put option is calculated in a similar way.

2.1 Black-Scholes Inputs

1. Stock Price Stock price is directly pulled off the market. The stock price used for any given is the value of the stock at the closing bell.

2. Strike Price Strike price is the price a derivative of an option is allowed to be exercised. This value is determined in the contract.

3. Time Until Maturity The time until maturity is measured in years; it is the amount of time left until the option is allowed to be exercised.



$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$P(S,t) = Ke^{-r(T-t)} - S + (SN(d_1) - Ke^{-r(T-t)}N(d_2)) = Ke^{-r(T-t)} - S + C(S,t)$$

4. Risk-Free Interest rate Risk free interest rate is the default investment return that can be made without risk. In the United States, this rate is often taken as the rate of government Treasury Bonds.

5. Volatility Volatility is a measure of the fluctuation of the stock option. The commonly used measure of volatility in the stock exchange is the beta value.

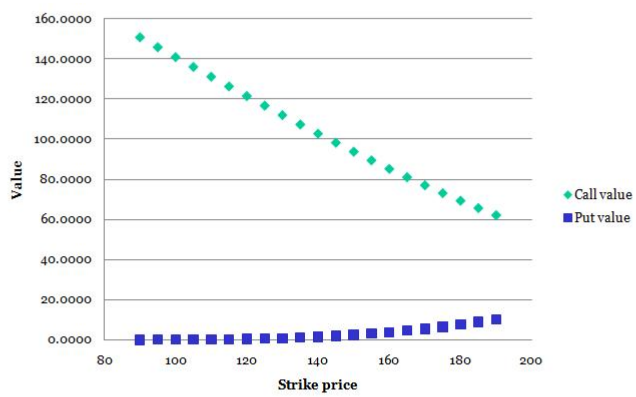
To obtain the inputs, data mining was used. Information for historical call and put options can easily be obtained in spreadsheet form on online financial data providers such as Yahoo! Finance. Using a macro, the data from the online source was imported into Excel. Then, with the help of open source coding, the Excel data was converted into a matrix in Java. These data points were then put into the Black-Scholes partial differential equations for analysis. The risk-free interest rate in the United States is obtained by consulting the government website for the current government T-bond rate. Volatility is a major part in stock prediction; it is also one of the most unpredictable factors in a stock market. In order to estimate future volatility, a separate model was constructed that calculated the volatility of the previous two days from historical data and used the obtained value as an estimator for the volatility of the next day. This data was imported into the Black-Scholes model from the volatility model.

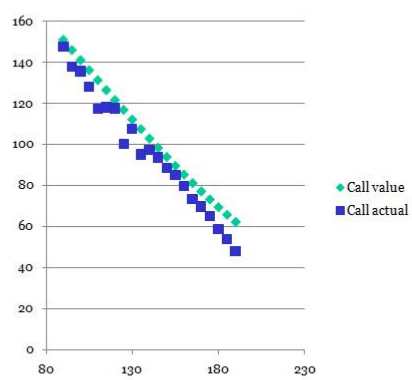
2.2 Functionality

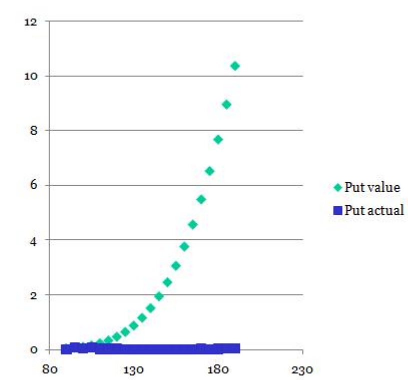
In order to utilize the Black-Scholes model, there are several assumptions that must be applied. For the cases studied in this paper, we assume accessibility to loan cash at the risk-free interest rate. This rate will be determined by the United States Treasury Bond rate, as it is the most accurate predictor of the risk-free rate in an American market. The valuation of options follows a Geometric Brownian motion and the return is normally distributed with no limits on shorting, no arbitrage, and no transaction costs or taxes. The company is assumed to not pay dividends (this assumption is often not true, but in these cases the dividend is often not large enough to greatly sway the results). Although in real life, the shares bought is discrete data, the Black-Scholes model disregards this aspect and treats all data as continuous.

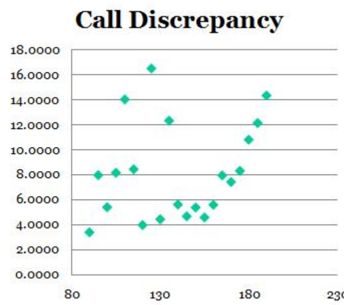
3 Results

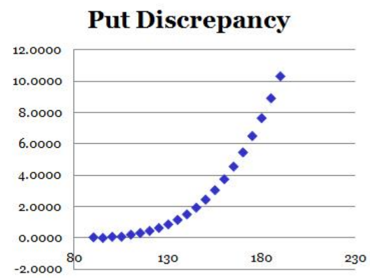
Although in the future many more companies will be analyzed, Apple (NASDAQ: AAPL) was used as a preliminary subject for analysis. At a given time t, the stock price for AAPL was 239.94.$APPLoptionsusedarerangedfrom$90.00 to 190.00$inincreasingincrementsof$5.00. All options were calculated with three days until maturity, volatility of 20The table below shows the results for the analysis for AAPL. Call value and put value columns denote the calculated values of call and put using the Black-Scholes model. Call actual and put actual columns denote real life data of the call and put values. The difference between call/put actual and call/put calculated is denoted in dCall and dPut. Both columns were then made weighed by dividing dCall and dPut by their respective expected values to calculate the error in both.











As we can see, the call values were much better predicted than the put values. Looking at the deviations between expected and actual values, the call deviations are small and randomly scattered (exhibit no curvilinear pattern), indicating that the model is a good fit. The put deviations, on the other hand, increase exponentially, indicating that the Black-Scholes model is not a good predictor of put values. The results obtained gave insight into future option pricing, as well as underline the main differences in American and European option trading. The results are presented in both table and graphical format (using spreadsheets and time-series plots). Although this is only one dynamic application of Black-Scholes, it may provide ideas for other investing tools that branch from models of markets of different nationalities.

3.1 Analysis and Discussion

Even if Black-Scholes is one of the most prevalent methods used in the investing world today, we can see from our results that the model does not always work. Why? First, if an option it out of the money (the strike price of the option is above the stock price), the option has no value. Applying a model to an option that has no value is similar to trying to register for negative five apples at the grocery store. The model is not fit to cater to out of the money options. Furthermore, the Black-Scholes model, like all models, cannot take into account factors outside of the stock market numbers. In a historical context, the B-S model cannot safeguard against stock market crashes or unexpected outside influences. Examples include the 9/11 terrorist attacks and the recent typo made by a trader on the NYSE.

3.2 Dangerous Assumptions

The Black-Scholes model comes with a set of assumptions. Sadly, these assumptions are far from valid in real life. As discussed above, the Black-Scholes model does not take into account extreme move. It assumes reasonable actions taken by the general public. In the example of the trader who typed one billion instead of one million, such an action would throw the model off. Another bad assumption that the model makes is instant, cost-free trading. In the real world, trading is far from instant. Even with high-speed technology, the time it takes for a trade to travel from the buyer (in any area of the country or even world) to a traders system on the floor of the stock exchange to the traders putting in of the order to the execution of the order, albeit miniscule, is enough for the market price to change. Stock prices are a continuum. Trading is also not cost-free. When buying and selling stocks, the trader has to pay a commission. When negotiating option contracts, there is a flat fee paid to establish the contract. In both cases, the fees must be considered when calculating profits. The Black-Scholes model relies on continuous time and continuous trading. However, the stock market is not 7-11, it does not stay open 24/7. In fact, it works an 8 hour workday (not even). There are large gaps (Friday afternoon to Monday morning), small gaps (Tuesday afternoon to Wednesday morning), and holidays (the stock exchange does not operate on Christmas). Within any of these time gaps when the market is closed, the price could change, and the opening price of one day may not necessarily match the closing price of the day before. The model does not take into account these differences. The last faulty of Black-Scholes is the assumption of standard trading. Standard trading requires that the currencies remain constant against each other, and the exchange rates are fixed. In the real world, currencies are also bought and sold on a daily basis on the FOREX, and their prices also fluctuate. When an assumption of the model is not met, there is an increase in risk. Unfortunately, when the model is used blindly without full understanding of these exterior risks, an investor can easily disregard the risks, making transactions seem more secure than they actually are. Many people point the ignorance of risk as a major contributor to economic crisis. When investments seem safer than they actually are, traders are more likely to overhedge and become overconfidence. This often leads to bubbles and extreme forms of speculation. Too often, the bubbles pop, creating a myriad of financial problems. (As Black-Scholes has progressed through the decades, there have been developments of a series of

Greek variables that account for these risks that are not explicitly included in the model. The calculation of these variables requires advanced mathematical tools and complicated financial data that are not generally accessible to the public. Therefore, these variables were not included in this study.)

4 Concluding Thoughts

When applying any model, and the Black-Scholes model is not an exception, we must keep in mind that models are just that, only models. Models can never predict human behavior with one hundred percent accuracy, nor can it replicate the intricacies of human behavior. The business of investment is not purely based on mathematics or theory. Investments are a complicated mix of math, perception, social behavior, and human emotion. To disregard any of these aspects would be a fatal mistake. While models are a good tool to use to aid investors, one should always exercise proper human judgment when dealing with trading. Never rely solely on models to make all the decisions, financial or otherwise. The human brain is still the best computer.