# An Analysis of a Dynamic Application of Black-Scholes in Option Trading

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April 9, 2010

**Abstract**

For decades people have invested in the stock market in with stocks, options, and bonds. Various groups of people have worked towards modeling the stock market with mathematics. One of the earliest is Black-Scholes. Developed by Fischer Black and Merton Scholes in 1973, it remains one of the most prevalent tools used by European investors today. However, the Black-Scholes model is catered toward European options, which have a definite time towards maturity. American stocks, on the other hand, do not have such constraints and can be bought and sold at any time. This project explores the ways in which Black-Scholes can be applied to the more dynamic American option trading market. The major focus of study will be comparing call and put values generated by the Black-Scholes model to historical call and put values.

**Keywords:** genetic algorithms, financial modeling

# 1 Introduction

## 1.1 Background

For decades people have invested in the stock market in with stocks, options, and bonds. Various groups of people have worked towards modeling the stock market with mathematics. One of the earliest is Black-Scholes. Developed by Fischer Black and Merton Scholes in 1973, it remains one of the most prevalent tools used by European investors today. The Black-Scholes involves several main variables: stock price, strike price, volatility, time until maturity, and the risk-free interest rate. Stock price denotes the current market price of the stock, and strike price denotes the price that the option can be exercised at. Time until maturity is the time until the option can be exercised; this value is often measured in years. The risk-free interest rate is the rate of return; in this experiment, the risk-free rate will be equal to the rate of a US Treasury Bond. Finally, volatility is the measure of variation in returns of a stock option. In the American stock exchange, volatility is often expressed in terms of beta.

## 1.2 Purpose

The purpose of this project is to investigate the Black-Scholes model, a popular tool in helping European investors determine the calls and puts of European options. The goal however, is to use the Black-Scholes model to estimate call and put values of American stock options. Another underlying purpose is to adapt the European model into an American model. Because unlike European options, which are held until a certain time (maturity), American options can be sold at any time, making the market much more dynamic. Conforming the model to American parameters can be a helpful investment tool to traders and investors alike.

## 1.3 Scope of Study

Most of the work revolves around the Black-Scholes model and the input of its required variables. Because the B-S model revolves around a constant time period, it will first be evaluated as such. The project will encorporate an excel model that takes the input of required values and generates call and put values from the input.

## 1.4 Methodology

The program will be coded in excel and then Java consisting of two main components. One is the stock class, holding all the required inputs. The other, perhaps the most important part of the project, is the Black-Scholes class containing the B-S formula and model. Outputs will be a series of numerical data. This data will then be outputted into a spreadsheet and graphed as a time-series plot. Inputs will be price, volatility, and interest rates. For testing purposes, pre-determined values will be used. However, historical data on stocks and options can also be used as inputs. To determine accuracy, the price can be compared to a calculator or historical data. The computation of each variable in the output will be a different sub-function, and each will be debugged and checked separately.

# 2 Project description

## 2.1 Black-Scholes Inputs

1. Stock Price Stock price is directly pulled off the market. The stock price used for any given is the value of the stock at the closing bell.

2. Strike Price Strike price is the price a derivative of an option is allowed to be exercised. This value is determined in the contract.

3. Time Until Maturity The time until maturity is measured in years; it is the amount of time left until the option is allowed to be exercised.

4. Risk-Free Interest rate Risk free interest rate is the default investment return that can be made without risk. In the United States, this rate is often taken as the rate of government Treasury Bonds.

5. Volatility Volatility is a measure of the fluctuation of the stock option. The commonly used measure of volatility in the stock exchange is the beta value.

## 2.2 Functionality

In order to utilize the Black-Scholes model, there are several assumptions that must be applied. For the cases studied in this paper, we assume accessibility to loan cash at the risk-free interest rate. This rate will be determined by the United States Treasury Bond rate, as it is the most accurate predictor of the risk-free rate in an American market. The valuation of options follows a Geometric Brownian motionand the return is normally distributed with no limits on shorting, no arbitrage, and no transaction costs or taxes. The company is assumed to not pay dividends (this assumption is often not true, but in these cases the dividend is often not large enough to greatly sway the results). Although in real life, the shares bought is discrete data, the Black-Scholes model disregards this aspect and treats all data as continuous. Black-Scholes calculates the value of call options using two differential equations. The put option is calculated in a similar way.

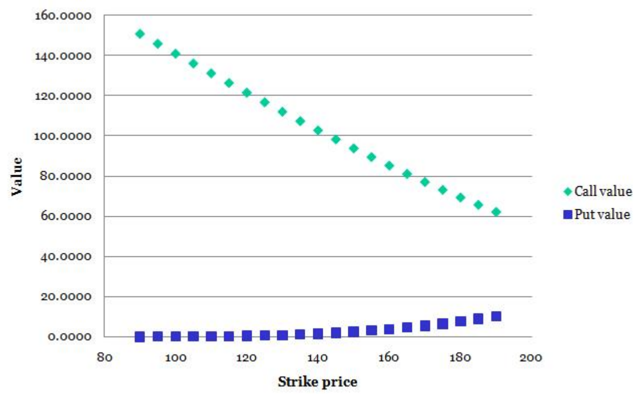
$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$
$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$
$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

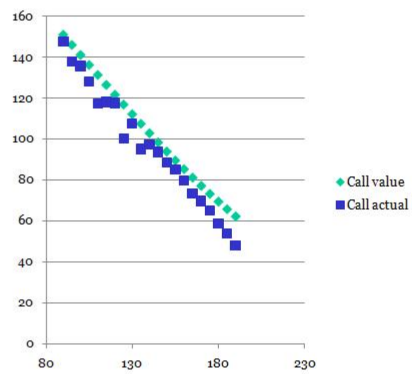
$$P(S,t) = Ke^{-r(T-t)} - S + (SN(d_1) - Ke^{-r(T-t)}N(d_2)) = Ke^{-r(T-t)} - S + C(S,t)$$

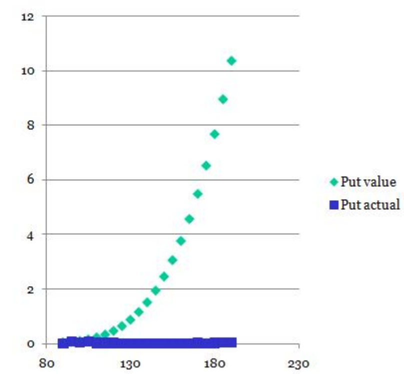
# 3 Results

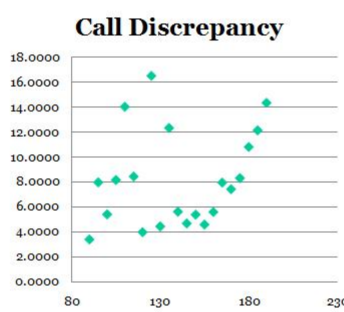
## 3.1 Apple Inc

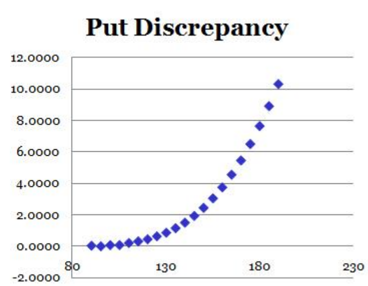
Although in the future many more companies will be analyzed, Apple (NASDAQ: AAPL) was used as a preliminary subject for analysis. At a given time t, the stock price for AAPL was 239.94. APPL options used are ranged from 90.00 to 190.00 in increasing increments of 5.00. All options were calculated with three days until maturity, volatility of 20The table below shows the results for the analysis for AAPL. Call value and put value columns denote the calculated values of call and put using the Black-Scholes model. Call actual and put actual columns denote real life data of the call and put values. The difference between call/put actual and call/put calculated is denoted in dCall and dPut. Both columns were then made weighed by dividing dCall and dPut by their respective expected values to calculate the error in both. The graph below compares the actual values in comparison to the calculated values. The weighted errors are shown in the following graph.




## 3.2 Further analysis

The results obtained should give insight into future option pricing, as well as underline the main differences in American and European option trading. The results will be presented in both table and graphical format (using spreadsheets and time-series plots). Although this is only one dynamic application of Black-Scholes, it may provide ideas for other investing tools that branch from models of markets of different nationalities.