An Analysis of Dynamic Applications of Black-Scholes

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Abstract

For decades people have invested in the stock market in with stocks, options, and bonds. One of the earliest is Black-Scholes. Developed by Fischer Black and Merton Scholes in 1973, it remains one of the most prevalent tools used by European investors today. The major focus of study will be comparing call and put values generated by the Black-Scholes model to historical call and put values.

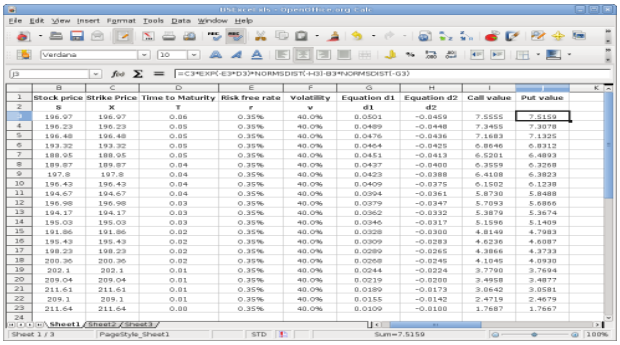


Figure 1: The Black-Scholes Spreadsheet

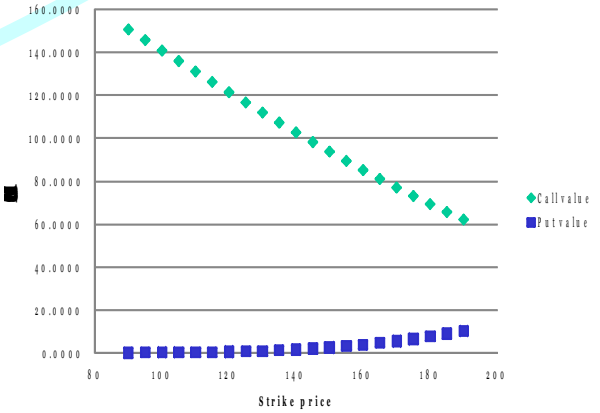
Figure 2: Main formulas of Black-Scholes

$$C(S,t) = SN(d_1) - Ke^{-r(T-t)}N(d_2)$$

$$d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

$$P(S,t) = Ke^{-r(T-t)} - S + (SN(d_1) - Ke^{-r(T-t)}N(d_2)) = Ke^{-r(T-t)} - S + C(S,t)$$



Modeling

The Black-Scholes involves several main variables: stock price, strike price, volatility, time until maturity, and the risk-free interest rate. We assume that the valuation of options follows a Geometric Brownian motionand the return is normally distributed with no limits on shorting, no arbitrage, no dividends, and no transaction costs or taxes. The volatility is calculated through a logarithmic function from historical data; the risk free rate is estimated by the U.S. T-bond rate. The generated values are then compared with actual values.

Findings

Although in the future many more companies will be analyzed, Apple (NASDAQ: AAPL) was used as a preliminary subject for analysis. At a given time t, the stock price for AAPL was $239.94. APPL options used are ranged from $90.00 to $190.00 in increasing increments of $5.00. All options were calculated with three days until maturity, volatility of 20%, and a risk free rate of 0.35%.