# TJ USAMO Practice 1

Varsity Math Team

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1. 51 numbers are to be chosen from 1, 2, 3, …, 99, 100. Show that no matter how these numbers are chosen, we can find two that have no common prime divisor.

2. Show that for all integers $n > 1$, the expression $n^4 + 4^n$ is not prime.

3. (IMO Shortlist 1986) $X$, $Y$, and $Z$ are points on the sides $BC$, $CA$, and $AB$ of triangle $ABC$ such that $AX$, $BY$, and $CZ$ concur at point $P$. Show that if $AYPZ$ and $BZPX$ are cyclic, then so is $CXPY$.

4. (IMO Shortlist 1996) Show that for all positive reals $x, y$, and $z$ such that $xyz = 1$, we have
$$\frac{xy}{x^5 + xy + y^5} + \frac{yz}{y^5 + yz + z^5} + \frac{zx}{z^5 + zx + x^5} \leq 1$$
When does equality hold?

5. Determine the largest positive integer $n$ with the property that any $2n \times 2n$ board covered with non-overlapping $1 \times 2$ tiles can be divided into two pieces with a straight cut without splitting any tiles.