

# TJ USAMO Practice 10 - Number Theory

Varsity Math Team

December 16, 2004

1. Determine all positive integers  $n$  for which

$$\frac{3n + 5}{7n + 11}$$

is reducible.

2. Prove that 121 does not divide  $n^2 + 3n + 5$  for any positive integer  $n$ .
3. Determine the remainder obtained when  $\sum_{k=0}^{p-1} n^k$  is divided by  $p$  where  $p$  is prime and  $n$  is an integer.
4. Determine all solutions, in integers, of the system

$$x + y + z = w \tag{1}$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{w} \tag{2}$$

5. Does there exist a positive integer that can be expressed as the sum of 1990 consecutive positive integers and expressed as the sum of such integers in exactly 1990 ways?