

TJ USAMO Practice 12 - Contest 2

Varsity Math Team

January 13, 2005

1. Show that on any tetrahedron we can find a vertex V such that the lengths of the 3 edges with V as an endpoint can be the sides of a triangle.
2. Let a , b , and c be non-zero integers such that both

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \quad \text{and} \quad \frac{a}{c} + \frac{c}{b} + \frac{b}{a}$$

are integers. Prove that $|a| = |b| = |c|$.

3. Let ABC be a scalene triangle, and P a point on the angle bisector of $\angle BAC$. Let the extensions of \overline{PB} and \overline{PC} intersect with the extensions of \overline{AC} and \overline{AB} at D and E respectively. Show that P lies on the circumcircle of ABC if and only if $EP \cdot DC = DP \cdot EB$.