

TJ USAMO Practice 6 - Geometric Transformations

Varsity Math Team

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1. A , B , C , and D are points on a line in that order such that $AB = CD$. Show that for any point P ,

$$AP + DP \geq BP + CP$$

(*Hint:* Use the following lemma: For all points Q on or inside triangle XYZ , $XY + XZ \geq QY + QZ$.)

2. Show that on the sides of any convex polygon P we can select three points A , B , and C such that ABC is equilateral. Now, generalize this and show that given any triangle DEF and smooth closed loop Γ , we can select three points X , Y , and Z on Γ such that XYZ is similar to DEF .
3. ABC is an acute triangle with $AB > AC$. D is the foot of the altitude from A to BC . P is a point on AD , and E and F the intersections of the extensions of BP and CP past P to AC and AB respectively. Show that DP bisects $\angle EDF$.