

TJ USAMO Practice 8 - Triangle Geometry

Varsity Math Team

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1. In triangle ABC , D , E , and F are the midpoints of \overline{BC} , \overline{CA} , and \overline{AB} respectively. \overline{AD} , \overline{BE} , and \overline{CF} concur at G . Determine the value of BC^2 in terms of AG , BG , and CG .
2. The incircle of ABC is tangent to \overline{BC} , \overline{CA} , and \overline{AB} at D , E , and F , respectively, and X , Y , and Z are the midpoints of \overline{EF} , \overline{FD} , and \overline{DE} respectively. Let I denote the incenter of triangle ABC . Line l_1 is the line passing through X that is parallel to \overline{ID} , and lines l_2 and l_3 are defined analogously with respect to Y and Z . Show that l_1 , l_2 , and l_3 are concurrent.
3. ABC is a triangle, oriented in the clockwise direction when traversed in the order listed, and P_0 is a point in the plane. Define the transformation $T : P_n \rightarrow P_{n+1}$ as follows: P_n is rotated 120 degrees counterclockwise about A , then 120 degrees counterclockwise about B , and finally 120 degrees counterclockwise about C , taking it to the new point P_{n+1} . Show that $P_0 = P_{1986}$ if and only if ABC is equilateral.
4. $\angle A$ is the smallest angle in the triangle ABC . Let U be an interior point of the arc of the circumcircle between B and C which does not contain A . The perpendicular bisectors of \overline{AB} and \overline{AC} meet the line AU at V and W , respectively. The lines BV and CW meet at T . Show that $AU = TB + TC$.