

Classic Problems

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This is a growing list of challenging problems that I've encountered. List 1.n consists of problems that, in my opinion, have very nice solutions. List 2.n contains challenging problems, and list 3.n contains problems that I have not solved yet. I have listed where the problems I borrowed come from. Some of these are extremely hard, so don't expect to solve them all too fast.

1.1 - Find all integers x less than 100 for which $x^4 - x^2 + 57$ is a multiple of 73.

1.2 - Compute the exact value of $\sum_{k=1}^{1001} \log_8 \left(\cos \frac{k\pi}{2003} \right)$

1.3 - ABCD is a quadrilateral with $AB = BC = 3$, and $CD = 7$. What is the maximum area that it could have?

1.4 (Po Shen 03') - Compute exactly: $\sum_{k=1}^{\infty} \tan^{-1} \left(\frac{1}{2k^2} \right)$.

1.5 - A beam of light is shined from the origin towards a flat mirror. It reflects off of the mirror at $(8, 4, 1)$ and then passes through $(10, 8, 5)$. What is the equation of the plane that contains the mirror? Express your answer in the form $x + Ay + Bz = C$.

1.6 - Given real numbers a , b , and c , such that $a, b, c > 0$, compute the minimum value of $\frac{a^3}{4b} + \frac{b}{8c^2} + \frac{1+c}{2a}$. Find all ordered triplets (a, b, c) that yield this minimal value.

2.1 (JGeneson 03') - Let $a_0 = 1$, and $a_n = 2a_{n-1} \sin \frac{n\pi}{48}$ for $n \geq 1$. Compute: $\prod_{i=1}^{47} a_i$.

2.2 - Compute the exact value of the expression: $\tan \frac{3\pi}{11} + 4 \sin \frac{2\pi}{11}$.