Solutions to Mock AIME 2

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1. Answer: **012**. The number of 2's in the prime factorization of 2004! is $\lfloor \frac{2004}{2^1} \rfloor + \lfloor \frac{2004}{2^2} \rfloor + \lfloor \frac{2004}{2^3} \rfloor + \cdots = 1002 + 501 + 250 + \cdots > 1000$. There are 2 2's in the prime factorization of 2004; hence $(2^2)^k | 2004!$ for all integers $k \leq 250$. Similarly, $3^k | 2004!$ for all integers $k \leq 200$, but there are only 12 167's in the prime factorization of 2004!. Hence, the answer is 12.

2. Answer: **843**. Notice that $\left(x^m + \frac{1}{x^m}\right)\left(x + \frac{1}{x}\right) = x^{m+1} + \frac{1}{x^{m+1}} + x^{m-1} + \frac{1}{x^{m-1}}$ so that $S_m S_1 = S_{m+1} + S_{m-1}$ or, equivalently, $S_{m+1} = S_m S_1 - S_{m-1}$. Therefore,

$$\begin{aligned} S_2 &= S_1S_1 - S_0 = 3 \cdot 3 - 2 = 7 \\ S_3 &= S_2S_1 - S_1 = 7 \cdot 3 - 3 = 18 \\ S_4 &= S_3S_1 - S_2 = 18 \cdot 3 - 7 = 47 \\ S_5 &= S_4S_1 - S_3 = 47 \cdot 3 - 18 = 123 \\ S_6 &= S_5S_1 - S_4 = 123 \cdot 3 - 47 = 322 \\ S_7 &= S_6S_1 - S_5 = 322 \cdot 3 - 123 = 843 \end{aligned}$$

ALTERNATE SOLUTION

Solve for $x = \frac{3 \pm \sqrt{5}}{2}$. These two values are reciprocals; WLOG we take $x = \frac{3+\sqrt{5}}{2}$ so that

$$\begin{aligned} S_7 = \left(\frac{3+\sqrt{5}}{2}\right)^7 + \left(\frac{3-\sqrt{5}}{2}\right)^7 &= \frac{(3+\sqrt{5})^7 + (3-\sqrt{5})^7}{2^7} \\ &= \frac{3^7 + \binom{7}{2} \cdot 3^5 \cdot 5 + \binom{7}{4} \cdot 3^3 \cdot 5^2 + \binom{7}{6} \cdot 3^1 \cdot 5^3}{2^6} \\ &= \frac{2187 + 21 \cdot 243 \cdot 5 + 35 \cdot 27 \cdot 25 + 7 \cdot 3 \cdot 125}{64} \\ &= \frac{2187 + 25515 + 23625 + 2625}{64} \\ &= \frac{53952}{64} = 843 \end{aligned}$$

3. Answer: **437**. Note that the probability we want is equivalent to the probability that among 6 balls drawn out simultaneously, no two have the same color. This can be accomplished only by choosing exactly one of each color, which leaves $4 \cdot 4 \cdot 2 \cdot 1 \cdot 1 \cdot 1$ possibilities out of $\binom{13}{6}$

total possibilities. Hence, the desired probability is

$$\frac{4\cdot 4\cdot 2\cdot 1\cdot 1\cdot 1}{\binom{13}{6}}=\frac{8}{429}$$

therefore, the answer is $8+429=437$.

4. Answer: **604**. We note that if 5^k has n digits and begins with 1, then 5^{k+1} has n digits and does not start with 1. If 5^k does not start with 1, then 5^{k+1} has $n+1$ digits. If every power of 5 starting at 5^1 started with a digit other than 1, then 5^k would have k digits. Since 5^{2004} has 1401 digits, we reason that 603 powers of 5 between 5^1 and 5^{2003} begin with 1. 5^{2004} begins with a 5, but we add in 1 for 5^0 and obtain the answer.

5. Answer: **255**. If a number has a 12-digit repeating decimal, its fractional part can be expressed as $\frac{abcdefghijkl}{999999999999}$, where each of $a-l$ is a digit 0 - 9. If this is to reduce to $\frac{1}{n}$, then $abcdefghijkl$ must divide $999999999999=10^{12}-1$. Hence, the possible n are $\frac{10^{12}-1}{abcdefghijkl}$. We factor:

$$\begin{aligned}10^{12}-1 &= \left(10^6+1\right)\left(10^6-1\right)\\ &= \left(10^2+1\right)\left(10^4-10^2+1\right)\left(10^3-1\right)\left(10^3+1\right)\\ &= 101\cdot 9901\cdot 9\cdot 111\cdot 11\cdot 91\\ &= 3^3\cdot 7\cdot 11\cdot 13\cdot 37\cdot 101\cdot 9901.\end{aligned}$$

Therefore, there are $(3+1)(1+1)^6=256$ factors of $10^{12}-1$, but one of these corresponds to $n=1$, which is disallowed. Hence, the answer is 255.

6. Answer: **351**. Since triangles AFP and FBP share an altitude from P, we have $\frac{BF}{FA}=\frac{[FBP]}{[AFP]}=\frac{1}{2}$. Let $[EAP]=k$. By similar reasoning, $\frac{AE}{EC}=\frac{k}{24}$. By Ceva's theorem, $\frac{CD}{DB}\frac{BF}{FA}\frac{AE}{EC}=1\implies\frac{CD}{DB}=\frac{48}{k}$. Now we note that $\frac{[ADC]}{[ABD]}=\frac{[PDC]}{[PBD]}=\frac{CD}{DB}=\frac{48}{k}$. Hence, $\frac{[ADC]-[PDC]}{[ABD]-[PBD]}=\frac{[APC]}{[APB]}=\frac{48}{k}$. We use the fact that $[APC]=[APE]+[EPC]=k+24$ and $[ABP]=[AFP]+[FBP]=126+63=189$. We have

$$\begin{aligned}\frac{24+k}{189} &= \frac{48}{k}\\ 48\cdot 189 &= k^2+24k\\ k &= \frac{-24\pm\sqrt{24^2+4\cdot 48\cdot 189}}{2}=-12\pm\sqrt{12^2+48\cdot 189}\\ &= -12\pm 12\sqrt{1+63}=-108,84\end{aligned}$$

We take $k=84$ since it represents an area. Now, $\frac{AE}{EC}=\frac{7}{2}$ and $\frac{CD}{DB}=\frac{4}{7}$. By Menelaus' theorem, $\frac{BF}{FA}\frac{AP}{PD}\frac{DC}{DB}=-1$ (Ceva and Menelaus use the convention of directed distances, where $XY=-YX$.) This yields $\frac{AP}{PD}=\frac{11}{2}$ from which $\frac{[ABPC]}{[PBC]}=\frac{11}{2}$. Hence, $[ABC]=\frac{13}{11}\cdot[ABPC]=\frac{13}{11}\cdot(24+84+126+63)=351$.

ALTERNATE SOLUTION

Assign the weights 1, 2, and ω to A, B, and C. It must be that $[EAP] = 24\omega, [DCP] = 2k$, and $[BDP] = \omega k$ for some k. But we have $\frac{2}{w} = \frac{[DCP]}{[BDP]} = \frac{[DCA]}{[BDA]} = \frac{[PCA]}{[BPA]} = \frac{24(\omega+1)}{126+3} = \frac{8(\omega+1)}{63}$. We solve this quadratic for $\omega = \frac{7}{2}, -\frac{9}{2}$, and choose the former since 24ω is an area. But the weight on D is $\omega + 2$ so that $\frac{\omega+2}{1} = \frac{AP}{PD} = \frac{[ABPC]}{[PBC]}$. Substituting, $\frac{11}{2} = \frac{24+24\cdot\frac{7}{2}+126+63}{[PBC]}$ which implies that $[PBC] = 54$. Therefore, $[ABC] = [ABPC] + [PBC] = 297 + 54 = 351$.

7. Answer: **000**. Note that if two complete suits are in the union Po-Ru and Reid's hands, then the other two complete suits are in Anders and Aaron's hands. There are $\binom{4}{2} = 6$ ways that the pairs of suits can be distributed. For each pair, one player has some 13 of the 26 cards, so the number of possible deals is $\binom{4}{2}\binom{26}{13}^2 = 6 \cdot \left(\frac{26\cdot25\cdot24\cdots15\cdot14}{13!}\right)^2$. Note that $\binom{26}{13}$ is divisible by 2 and 25, hence N is divisible by $2 \cdot (2 \cdot 25)^2 = 5000$. Therefore, the last three digits of N are 000.

8. Answer: **704**. Obviously, the last three digits of 2004^k are the same as 4^k. It is also clear that $k = 2003^{2002^{2001}}$ exceeds 2, so that $4^{2003^{2002^{2001}}} \equiv 0 \pmod 8$. Let us determine $4^{2003^{2002^{2001}}} \pmod{125}$.

   Because $\phi(125) = 100$, we have $4^{2003^{2002^{2001}}} \equiv 4^{3^{2002^{2001}}} \pmod{125}$. We are interested in $3^{2002^{2001}} \pmod{100}$.

   We play the same card again, that is, $\phi(100) = 40$ so that $3^{2002^{2001}} \equiv 3^{2^{2001}} \pmod{100}$. We are also interested in $2^{2001} \pmod{40}$. Clearly, 8 divides 2^{2001} so that $2^{2001} \equiv 0 \pmod 8$. We also have $2^{2001} \equiv 2 \pmod 5$ by Fermat's little theorem. By the Chinese remainder theorem, it must be that $2^{2001} \equiv 32 \pmod{40}$. Now we need $3^{32} \pmod{100}$. This can be quickly found: $3^4 = 81 \Longrightarrow 3^8 \equiv (81)^2 \equiv 61 \pmod{100} \Longrightarrow 3^{16} \equiv (61)^2 \equiv 61^2 \equiv 21 \pmod{100} \Longrightarrow 3^{32} \equiv (21)^2 \equiv 41 \pmod{100}$.

   Therefore, $4^{2003^{2002^{2001}}} \equiv 4^{41} \pmod{125}$. $2^7 = 128 \equiv 3 \pmod{125}$, hence $4^{41} \equiv 2^{82} \equiv 3^{11}\cdot 2^5 \pmod{125}$. $3^7 = 2187 \equiv 62 \pmod{125}$ so that $3^{11}\cdot2^5 \equiv 81\cdot32\cdot62 \equiv 79 \pmod{125}$. We apply the Chinese remainder theorem again, and determine that the unique residue r such that $r \equiv 0 \pmod 8$ and $r \equiv 79 \pmod{125}$ is $r \equiv 704 \pmod{1000}$.

9. Answer: **280**. It is known that $3^{k+1} > 3^k + 3^{k-1} + \cdots + 3 + 1$. In the expansion of the product, each k_i consists only of 1's and 0's when written in trinary since the exponent is compiled from distinct powers of three from certain binomials. Since $1997 = 11111001101_2$, we have $k_{1997} = 111110011010_3$. It follows that $a_{1997} = 1\cdot3\cdot4\cdot7\cdot8\cdot9\cdot10\cdot11$. This can be multiplied out (obtaining 665,280) or kept modulo 1000; either method done correctly gives a remainder of 280 under division by 1000.

10. Answer: **727**. Pythagoras gives $AN = 20$. We draw BD and AD, and construct the altitute MP to AD, with P on AD, and altitude MM' to AE, with M' on AE. Because $BC = CD = DE$, angles BAC, CAD, and DAE are congruent. Because P is on AD, triangles MNA and MPA are congruent by AAS, so $MP = 15$ and $PA = 20$, from which Pythagoras gives $PD = 112$, implying $AD = 132$.

    Let $\alpha = m\angle BAC$, so $m\angle MAE = 2\alpha$, and $m\angle NAE = 3\alpha$. Because we have $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$, we compute $\sin(2\alpha) = \frac{24}{25}$, and $\sin(3\alpha) = \frac{117}{125}$. We find that $MM' = 24$ using $\sin(2\alpha) = \frac{24}{25}$. By a simple Law of Sines argument $DE : EB : BD = 25 : 39 : 40$.

Let $[ABE]$ = the area of ABE. We have $[ABE] = 1/2(15 \cdot AB + 24 \cdot AE)$.

Ptolemy on $ABDE$ yields $AB \cdot DE + AE \cdot BD = AD \cdot BE$. Using the abundance of facts that we have ascertained previously, this gives:

$$\begin{aligned} AB \cdot 25x + AE \cdot 40x &= 132 \cdot 39x \\ 25AB + 40AE &= 39 \cdot 132 \\ 15AB + 24AE &= \frac{39 \cdot 132 \cdot 3}{5} \end{aligned}$$

Finally, $[ABE] = \frac{1}{2} \cdot (15AB + 24AE) = \frac{1}{2}\frac{39 \cdot 132 \cdot 3}{5} = \frac{7722}{5}$. Therefore, the answer is $722 + 5 = 727$.

11. Answer: **167**. It is known that $\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$. If we set $A = \tan^{-1}(a)$ and $B = \tan^{-1}(b)$, then we may write $\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$. Using this, we may write

$$\begin{aligned} \omega &= \alpha + \beta + \gamma \\ &= \tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right) + \tan^{-1}(\gamma) \\ &= \tan^{-1}\left(\frac{\left(\frac{\alpha+\beta}{1-\alpha\beta}\right) + \gamma}{1 - \left(\frac{\alpha+\beta}{1-\alpha\beta}\right)\gamma}\right) \\ &= \tan^{-1}\left(\frac{\alpha+\beta+(1-\alpha\beta)\gamma}{(1-\alpha\beta)-\gamma(\alpha+\beta)}\right) \\ &= \tan^{-1}\left(\frac{\alpha+\beta+\gamma-\alpha\beta\gamma}{1-(\alpha\beta+\beta\gamma+\gamma\alpha)}\right) \\ \implies \tan(\omega) &= \frac{\alpha+\beta+\gamma-\alpha\beta\gamma}{1-(\alpha\beta+\beta\gamma+\gamma\alpha)} \end{aligned}$$

We expand the given equation, obtaining $4x^3 - 799x^2 - 200x - 1 = 4(x-\alpha)(x-\beta)(x-\gamma) = 0$. We have $\alpha + \beta + \gamma = \frac{799}{4}$, $\alpha\beta + \beta\gamma + \gamma\alpha = -50$, and $\alpha\beta\gamma = \frac{1}{4}$. Therefore, $\tan(\omega) = \frac{\frac{799}{4} - \frac{1}{4}}{1+50} = \frac{399}{102} = \frac{133}{34}$. It follows that the answer is $133 + 34 = 167$.

12. Answer: **589**. Consider D' on the circumcircle of $ABCD$ such that $CD' = 7$ and $D'A = 1$. Let $m\angle D'AB = \alpha$ and $m\angle BCD' = \pi - \alpha$. Then by the Law of Cosines,

$$\begin{aligned} 1^2 + 8^2 - 2 \cdot 1 \cdot 8\cos(\alpha) &= BD'^2 = 4^2 + 7^2 - 2 \cdot 4 \cdot 7\cos(\pi - \alpha) \\ \implies \cos(\alpha) &= 0 \end{aligned}$$

Hence $D'AB$ is a right triangle and the circumradius of $ABCD$ is $\frac{\sqrt{65}}{2}$. Now, by similar triangles, we have $AP : BP : CP : DP = 56 : 32 : 4 : 7$. Let $AP = 56x$ so that $AC = 60x$ and $BD = 39x$. Ptolemy's theorem applied to $ABCD$ yields $60x \cdot 39x = 1 \cdot 8 + 4 \cdot 7 = 36$ from which $x^2 = \frac{1}{65}$.

Now we apply Stewart's theorem to triangle BOD and cevian OP, obtaining

$$\begin{aligned} OB^2 \cdot PD + OD^2 \cdot BP &= OP^2 \cdot BD + BP \cdot BD \cdot PD \\ \frac{65}{4}(32x + 7x) &= 39x \cdot OP^2 + 32x \cdot 39x \cdot 7x \\ \frac{65}{4} - 7 \cdot 32\frac{1}{65} &= OP^2 \\ OP^2 &= \frac{3329}{260} \end{aligned}$$

It follows that the answer is $329 + 260 = 589$.

13. Answer: **071**. Consider the polynomial $Q(x) = x \cdot (x + 1) \cdot P(x) - 1$. The given implies that $Q(x) = 0$ for $x = 1, 2, 3, \cdots, 10$. Therefore, we may write $Q(x) = R(x)(x - 1)(x - 2)(x - 3)\cdots(x - 10)$ for some polynomial $R(x)$. But $Q(0) = Q(-1) = -1$, so that $R(x)$ is non-constant. Hence, the minimum degree $Q(x)$ that corresponds to the minimum degree $P(x)$ must be of the form $(ax + b)(x - 1)(x - 2)\cdots(x - 10)$,. Setting $x = 0$, we find that $-1 = 10! \cdot b \iff b = \frac{-1}{10!}$. Setting $x = -1$ yields $-1 = 11! \cdot (b - a) \iff a = b + \frac{1}{11!} = \frac{-10}{11!}$. Therefore, $Q(11) = \left(11 \cdot \frac{-10}{11!} + \frac{-1}{10!}\right) \cdot 10 \cdot 9 \cdot 8 \cdots 1 = -11 = 11 \cdot 12 \cdot P(11) - 1$. We solve for $P(11) = \frac{-10}{132} = \frac{-5}{66}$ from which it follows that the answer is 71.

ALTERNATE SOLUTION

Define $\Delta^k(n) = \Delta^{k-1}(n + 1) - \Delta^{k-1}(n)$ where $\Delta^0(n) = P(n)$. We argue that $\Delta^k(n) = \frac{(-1)^k(k+1)!}{n(n+1)\cdots(n+k+1)}$ for all positive integers n and k for which $n + k \leq 10$. We induct on k; obviously the base case $k = 0$ is true. If we assume this identity for row k, then

$$\begin{aligned} \Delta^{k+1}(n) &= \Delta^k(n + 1) - \Delta^k(n) = \frac{(-1)^k(k + 1)!}{(n + 1)(n + 2)\cdots(n + k + 2)} - \frac{((-1)^k(k + 1)!}{n(n + 1)\cdots(n + k + 1)} \\ &= \frac{(-1)^k(k + 1)!}{(n + 1)(n + 2)\cdots(n + k + 1)} \cdot \left(\frac{1}{n + k + 2} - \frac{1}{n}\right) \\ &= \frac{(-1)^{k+1}(k + 1)!}{(n + 1)\cdots(n + k + 1)}\left(\frac{k + 2}{n(n + k + 2)}\right) \\ &= \frac{(-1)^{k+1}(k + 2)!}{n(n + 1)\cdots(n + k + 2)} \end{aligned}$$

The first k such that $\Delta^k(n)$ is constant for all integers n must be at least $k = 9$; hence P is at least 9th degree. Since P is of minimal degree, we may assert that $\Delta^9(n)$ is constant. We may now retrace our subtractions to find $\Delta^0(11)$. Specifically,

$$\begin{aligned} \Delta^0 P(11) &= \Delta^0(10) + \Delta^1(10) = \Delta^0(10) + \left(\Delta^1(9) + \Delta^2(9)\right) = \cdots \\ &= \sum_{k=0}^{9} \Delta^k(10 - k) = \sum_{k=0}^{9} \frac{(-1)^k(k + 1)!}{(10 - k)(11 - k)\cdots(11)} \\ &= \frac{1!}{10 \cdot 11} - \frac{2!}{9 \cdot 10 \cdot 11} + \frac{3!}{8 \cdot 9 \cdot 10 \cdot 11} - \cdots - \frac{10!}{11!} \end{aligned}$$

$$
\begin{aligned}
&= \frac{1!9! - 2!8! + 3!7! - \cdots - 10!0!}{11!} \\
&= \frac{5!(3024 - 672 + 252 - 144 + 120 - 144 + 252 - 672 + 3024 - 30240)}{11!} \\
&= \frac{-25200}{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} \\
&= \frac{-5}{66}
\end{aligned}
$$

From which it follows that the answer is $5 + 66 = 71$.

14. Answer: **152**. We will tackle the analgous problem of a string of 12 characters consisting of 3 E's, 4 D's, and 5 O's such that no two E's are adjacent and no D is next to an O. Write the three E's, that is, we consider 1E2E3E4. We must separate the E's with solid blocks of D's and O's in slots 2 and 3, but slots 1 and 4 can be empty or contain a solid block. We consider three cases.

    Case I - 1 and 4 are blank. Then either 2 is 4 D's and 3 is 5 O's or vice versa. There are two possible arrangements.

    Case II - 1 is not blank, but 4 is blank. Since for each of these arrangements, we could swap 1 and 4, we need not consider the case 1 blank and 4 not-blank separately, and merely double the number of strings in this case. Either there are 2 blocks of D's and 1 of O's or 1 block of D's and two of O's. In the first subcase, there are 3 ways to choose slots according to type, and 3 ways to distribute the 4 D's among two non-empty slots. In the latter subcase, there are again three ways to choose slot types but there are 4 ways to distribute 5 O's into two non-empty slots. Hence there are $9 + 12 = 21$ strings in this case, but via the bijection we count this as 42.

    Case III - 1 and 4 are both non-blank. If three of the four slots contain only D's, there are 4 type arrangements and 3 quantity arrangements for a total of 12 possibile strings. If exactly two of $\{1, 2, 3, 4\}$ are D's and the other two are O's, then there are 6 possible type arrangements. $x + y = n$ has $n - 1$ solutions in positive integers, hence this subcase has $6 \cdot 3 \cdot 4 = 72$ possible strings. Finally, if there is one slot filled with D's and three filled with O's, then there are 4 type arrangements. $x+y+z = n$ has $\binom{n-1}{2}$ solutions in positive integers, so this gives $4 \cdot \binom{4}{2} = 24$ strings. Adding, we have $12 + 72 + 24 = 108$ such strings.

    Therefore, the answer is $2 + 42 + 108 = 152$.

15. Answer: **529**. Drop altitude AA'. We have $m\angle AA'B = \frac{\pi}{2} - B$, but AOB is an isosceles triangle with $m\angle AOB = 2C \iff m\angle BAO = \frac{\pi}{2} - C$. Therefore, $\cos DAA' = \cos(C - B)$. Therefore we have $AD \cos(C - B) = AA' = AC \sin(C) = 2R \sin(B) \sin(C)$ so that $\frac{2R}{AD} = \frac{\cos(C-B)}{\sin(B)\sin(C)}$. Now,

    $\frac{2R}{AD} + \frac{2R}{BE} + \frac{2R}{CF} = \frac{\cos(C-B)}{\sin(B)\sin(C)} + \frac{\cos(A-C)}{\sin(C)\sin(A)} + \frac{\cos(B-A)}{\sin(A)\sin(B)}$
    $\iff 2R\sin(A)\sin(B)\sin(C)\left(\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF}\right)$
    $= \sin(A)\cos(B - C) + \sin(B)\cos(C - A) + \sin(C)\cos(A - B)$
    $= 3\sin(A)\sin(B)\sin(C) + \sin(A)\cos(B)\cos(C) + \sin(B)\cos(A)\cos(C) + \sin(C)\cos(A)\cos(B)$
    $= 3\sin(A)\sin(B)\sin(C) + \sin(A + B)\cos(C) + \sin(C)\cos(A)\cos(B)$

$= 3\sin(A)\sin(B)\sin(C) + \sin(C)\left(\cos(C) + \cos(A)\cos(B)\right)$
$= 3\sin(A)\sin(B)\sin(C) + \sin(C)\left(-\cos(A+B) + \cos(A)\cos(B)\right) = 4\sin(A)\sin(B)\sin(C)$
$\Longrightarrow \frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{R}$

Heron's formula yields $[ABC] = \sqrt{45 \cdot 5 \cdot 8 \cdot 32} = 240$. We substitute this into $[ABC] = \frac{abc}{4R} \iff R = \frac{abc}{4 \cdot [ABC]} = \frac{13 \cdot 37 \cdot 40}{4 \cdot 240} = \frac{13 \cdot 37}{24}$. From this we find that

$$\frac{1}{AD} + \frac{1}{BE} + \frac{1}{CF} = \frac{2}{R} = \frac{48}{481}$$

It follows that the answer is $48 + 481 = 529$.

1 Footnotes

The spectacular identity that we found in #15 was the goal of Iberoamerican 1985/B3. Problems #4, #6, and #11 were inspired by AIME 1990/4, AIME 1985/6, and Iberoamerican 1987/B2 respectively. Problem #4 was possible thanks largely due to the copy Mathematica 5.0 for Students that I received at MOSP 2004. Intriguingly, extraneous information was given in the 1985 AIME statement, which gave a fourth area in a setup like problem #4 of this contest. Problem #9 was submitted to me by Fermatprime.