

# TJMC #2

NO CALCULATORS, 40 Minutes

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2.1 - John has a biscuit. Joe asks John if he can have half of his biscuit. John says ok, and gives him “half” of his biscuit. Joe complains, saying “Your half is bigger than mine.” John agrees, and tearing off a third of his “half” of the biscuit and eating it. John then shows that the two new “halves” are now equal. What fraction of the biscuit did John give to Joe?

2.2 - How many zeros are at the end of 2003!?

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2.3 - How many integers between 1 and 1,000,000,000,000 inclusive are perfect squares, perfect cubes, or an integer raised to the fourth power?

2.4 - Let  $f(x)$  be a quadratic function passing through points  $(1, 10)$ ,  $(2, 0)$  and  $(3, -5)$ . If  $g(x) = a(x - 1)(x - 2) + b(x - 1)(x - 3) + c(x - 2)(x - 3)$ , and  $g(x) = f(x)$  for all  $x$ , compute the numerical value of  $a + b + c$ .

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2.5 - Points A through H are selected such that:

- i*) Angles ABC, CAD, DAE, AEF, AHF, and HFG are all right angles.
- ii*)  $AB = 3$ ,  $BC = 4$ ,  $CD = 13$ ,  $DE = 20$ ,  $FE = 20$ ,  $AH = 31$ , and  $GH = 15$ .

Compute the numerical value of  $GF^2$ .

2.6 - A polynomial with integer coefficients passes through the points  $(-2, 1)$  and  $(2003, k)$ . What is the largest possible  $k$  such that  $k \leq 100,000$ ?

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2.7 - Determine all real  $x$  such that:

$$x = \frac{3}{\frac{3}{x} + 3 + \frac{x}{3}} + 3 + \frac{\frac{3}{x} + 3 + \frac{x}{3}}{3}$$

2.8 - ABCD is a cyclic quadrilateral inscribed in circle O. AB is extended beyond B, and DC is extended beyond C. AB and DC intersect at P. A point M is selected on O such that MD = 4 and MC = 9. Another point N is selected such that NC = 9. Given that AB = 4, and that PC + 1 = PB = DC - 1, find the sum of the two possible lengths of ND.