TJMC #3

NO CALCULATORS, 40 Minutes

3.1 - In Puzzletopia, there are two kinds of people, those who always lie, known as Fibsters, and those who always tell the truth, known as Beholders. At a big meeting of 2003 of Puzzletopians, one of them says, "We are all Fibsters." A second says, "One of us is a Beholder." A third Puzzletopian says, "No, two of us are Beholders." This goes on until the last one says "All but one of us are Beholders." Given that at least one of them is telling the truth, how many Fibsters are there?

3.2 - A regular octagon has a perimeter of 80. What is its area?

3.3 - What is the lowest degree that a polynomial passing through the points $(0, 4)$, $(1, 1)$, $(2, 0)$, $(3, 1)$, and $(4, 4)$ can have?

3.4 - How many positive integers satisfy both of the following:

i) All of the digits are either 1, 2, 3, or 4.
ii) The sum of the integer's digits is 8.

3.5 - $ABCD$ is a tetrahedron such that $AB = 11, AC = 12, AD = 13, BC = 14, CD = 15,$ and $DB = 16$. Let M be the midpoint of AB and N be the midpoint of CD. Compute the numerical value of MN^2.

3.6 - The sum

$$\sum_{k=1}^{2003} kcis\left(\frac{360k^o}{2003}\right)$$

Can be expressed in the form $A + Bi$. Compute A. Note $cis\alpha = cos\alpha + isin\alpha$, where $i = \sqrt{-1}$.

3.7 - Find all positive integral x such that $x^4 - x^2 + 9012345$ is a multiple of 73.

3.8 - Three circles ω_1, ω_2, and ω_3 are externally tangent and have radii of $\frac{1}{5}, \frac{1}{7}$, and $\frac{1}{11}$ respectively. Two other circles Ω_1, and Ω_2 are each tangent to all three ω_n. Specifically, the radius of Ω_2 is greater than the radius of Ω_1. Compute the sum of the radii of Ω_1 and Ω_2.