

# TJMC #4

NO CALCULATORS, 40 Minutes

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4.1 - Triangle ABC has an area of 2003. Let D, E, and F be the midpoints of BC, AC, and AB respectively. Compute the area of  $\triangle DEF$ .

4.2 - There is one two digit positive integer that is 3 more than a multiple of 2, 4 more than a multiple of 3, 5 more than a multiple of 4, and 6 more than a multiple of 5. Find this number.

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4.3 - Find the number of real ordered pairs  $(x, y)$  such that both of the following are satisfied:

$$\begin{aligned} i) & x^2 + 4y = 4 \\ ii) & y^2 + 2x = 16 \end{aligned}$$

4.4 -  $f(x) = x^3 - 21x^2 + 15x - 8$  is a polynomial with three roots  $\omega_1, \omega_2$ , and  $\omega_3$ . Compute the numerical value of  $(\omega_1)^3 + (\omega_2)^3 + (\omega_3)^3$ .

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4.5 -  $a_1, a_2, \dots$  are real numbers such that  $n^2(a_1 + a_2 + \dots + a_n) = 10 + 30(a_{n+1} + a_{n+2} + \dots + a_{30})$  for positive, integral  $n$ . Compute  $a_{14}$ .

4.6 - Thomas is walking in the coordinate plane. Starting at the origin, he walks to  $(6, 0)$ . He then turns left  $\alpha$  degrees and then walks a fraction  $x$  of the distance he just walked to get to  $(9, 4)$ . He turns left  $\alpha$  degrees again and walks  $x$  of the distance he walked to get to another point. This process of turning and walking a smaller amount continues on and on. Thomas eventually begins to converge on a point. Find this point.

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4.7 - Find all real  $x$  for which  $x^5 + 5x^4 - 20x^2 - 2x + 14 = 0$ .

4.8 - Four spheres  $\Omega_1, \Omega_2, \Omega_3$ , and  $\Omega_4$  are of radius 4, 6, 6, and 4 respectively.  $\xi$  is tetrahedron is formed with its vertices at the centers of each  $\Omega_n$ . What is the volume of  $\xi$ ?