

# TJ USAMO Practice 12

VMT Math Team

February 15, 2004

1. There is a basketball machine that has a  $(p + 1)/(p + q + 2)$  probability of making a given shot where  $p$  and  $q$  are the number of shots made and missed previously to the current shot, respectively. Show that the probability of making  $k$  shots out of  $N$  attempts is equal for all  $0 \leq k \leq N$ .
2. (USAMO 99) A set of  $N > 3$  real numbers has sum at least  $n$  and the sum of the squares of the numbers is at least  $n^2$ . Show that the largest positive number is at least 2.
3. (USAMO 96)  $D$  lies inside the triangle  $ABC$ , and  $\angle BAC = 50^\circ$ ,  $\angle DAB = 10^\circ$ ,  $\angle DCA = 30^\circ$ ,  $\angle DBA = 20^\circ$ . Show that  $\angle DBC = 60^\circ$ .