# TJ USAMO Practice 14

VMT Math Team

March 7, 2004

1. (USAMO 1979) A certain organization has $n$ members, and it has $n+1$ three-member committees, no two of which have identical membership. Prove that there are two committees which share exactly one member.

2. (MOP 03) Show that every $2^n \times 2^n$ board with one square removed can be covered by Triominoes.

3. (Titu98) Let $a$ be a real number such that $\sin a + \cos a$ is a rational number. Prove that for all $n \in \mathbb{N}$, $\sin^n a + \cos^n a$ is rational.

4. (Titu98) Let $f : \mathbb{N} \to \mathbb{N}$ be such that $f(n+1) > f(f(n))$ for all $n \in \mathbb{N}$. Prove that $f(n) = n$ for all $n$.

5. Suppose that $P_1P_2 \ldots P_{325}$ is a regular 325-sided polygon. Determine the number of noncongruent triangles of the form $P_iP_jP_k$ where $i, j$, and $k$ are distinct integers between 1 and 325, inclusive.