TJ USAMO Practice 15

VMT Math Team

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1. a, b, and c are postive real numbers. Show that

$$\frac{2a+b+c}{a+2b+2c}+\frac{2b+c+a}{b+2c+2a}+\frac{2c+a+b}{c+2a+2b}\geq\frac{12}{5}$$

2. Show that for any triangle ABC, the inradius r is at most half of the circumradius R. (*Hint*: For any triangle, $1+\frac{r}{R}=\cos A+\cos B+\cos C$.)

3. Let S be the set $\{1, 2, 3, 4, 5\}$. Determine the value of

$$\sum_{a,b,c,d\in S}\left\lfloor\frac{(ac+bd)^2}{(a^2+b^2)(c^2+d^2)}\right\rfloor$$

4. a, b, c, and d are positive reals such that $c^2+d^2\geq(a^2+b^2)^3$. Show that

$$\frac{a^3}{c}+\frac{b^3}{d}\geq 1$$