TJ USAMO Practice 17 - Contest

April 12, 2004

4-6pm EST

1. Show that for any positive reals a, b, c,

   $$(a^4+b^4+c^4)(a^6+b^6+c^6)(a^8+b^8+c^8) \geq (a^2b^4+b^2c^4+c^2a^4)(a^3b^3+b^3c^3+c^3a^3)(a^4b^2+b^4c^2+c^4a^2)$$

2. Prove that for any positive integers n and k there exists a set $S = \{s_1, \ldots, s_n\}$ with the property that $(s_i - s_j)^k | s_i s_j$ for any integers $1 \leq i < j \leq n$.

3. ω_1 and ω_2 are concentric circles such that the area of ω_2 is four times as large as the area of ω_1. A, B, C, and D are four points (in that order) on ω_1. $\overline{AB}, \overline{BC}, \overline{CD}$, and $\overline{DA}$ are extended (past A, B, C, and D respectively) to E, F, G, and H (again, respectively) on ω_2. Show that the perimeter of $EFGH$ is at least twice that of $ABCD$, and determine when equality holds.