TJ USAMO Practice 5

VMT Math Team

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1. Prove that for all $a, b, c > 0$

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$$

2. (MOP 1998) Prove that for $x, y, z > 0$

$$\frac{x}{(x+y)(x+z)} + \frac{y}{(y+z)(y+x)} + \frac{z}{(z+x)(z+y)} \leq \frac{9}{4(x+y+z)}$$

3. Prove that for positive real numbers a, b, c with $a + b + c = 1$,

$$a^3 + b^3 + c^3 \geq \frac{1}{9}$$

4. (MOP 2003) Prove that for all nonzero $a, b, c \in \mathbb{R}$,

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} \geq \frac{a}{c} + \frac{c}{b} + \frac{b}{a}$$

5. (Titi97) Prove that for positive $x_1, x_2, \ldots, x_n$,

$$\frac{x_1^3}{x_1^2 + x_1x_2 + x_2^2} + \frac{x_2^3}{x_2^2 + x_2x_3 + x_3^2} + \cdots + \frac{x_n^3}{x_n^2 + x_nx_1 + x_1^2} \geq \frac{x_1 + x_2 + \cdots + x_n}{3}$$