

Triangle Geometry Problem Set

2003

1. Right triangle ABC with right angle C has sidelengths $AC = 3$ and $BC = 4$. Altitude CD is constructed, with D on the hypotenuse of ABC . Compute the length of CD .
2. In scalene triangle ABC , D is the midpoint of BC , E is the midpoint of AC , and F is the midpoint of AB . The area of triangle DEF is 6. Compute the area of triangle ABC .
3. In triangle ABC , $AB = 13$, $BC = 14$, and $CA = 15$. Altitudes AD and BE are constructed. Compute the length of ED .
4. Isosceles triangle ABC has $AC = BC$. Let D be on side BC such that AD is the angle bisector of angle BAC . If $AB = CD = 10$, what is the length of side AC ?
5. (AIME) Points D_1 and D_2 , E_1 and E_2 , and F_1 and F_2 are on sides BC , AC , and AB (pairwise respectively) of triangle ABC , such that D_1E_2 , E_1F_2 , and F_1D_2 are concurrent at P and are parallel to sides AB , BC , and AC respectively. If $[D_1D_2P] = 8$, $[E_1E_2P] = 18$, and $[F_1F_2P] = 50$, what is the area of triangle ABC ?
6. In triangle ABC , $AB = 3$ and $AC = 2$. D_1 is chosen on BC and D_2 on D_1C such that angles BAD_1 , D_1AD_2 , and D_2AC are congruent. If $BD_1 = 2$ and $D_2C = 1$, what is the maximum possible area of ABC ?
7. Triangle ABC has sidelengths $AB = 13$, $AC = 14$, $BC = 15$. Triangle $A_1B_1C_1$ lies outside triangle ABC and has sides parallel to ABC with a distance of 2 between corresponding sides. Compute the area of triangle $A_1B_1C_1$.
8. (ARML) A , B , and C are points on a line in that order, with $AB = 2$ and $BC = 9$. Points E and F are chosen on the same side of this line such that $AE = 3$, $EC = 10$, $BF = 33$, and $CF = 30$. Let the intersection of BF and CE be D . The value of the expression $\frac{[ABDE]}{[CDF]}$ can be expressed as $\frac{p}{q}$, where p and q are relatively prime, positive integers. Compute $p + q$.
9. (EDruker) In acute triangle ABC , the angle bisector of $\angle BAC$ intersects BC at K and meets the circumcircle of ABC again at P . The foot of the perpendicular from K to AC is N . Given $KP = 15$, $AN = 20$, and $AK = 25$, compute the area of triangle ABC .

10. In triangle ABC , the incircle, which has a radius of 5, is tangent to sides AB , AC , and BC at Z , Y , and X respectively. The lines AX , BY , and CZ concur at P . If $\frac{5AP}{XP} + \frac{5BP}{YP} + \frac{5CP}{ZP} = 74$, and $2 \cos B = \cos A + \cos C$, compute the area of ABC .