

Trigonometry Problem Set

2003

1. $\cos x = \frac{4}{5}$. Find all possible values of $\sin x$ and $\tan x$.
2. $\sin x + \cos x = \frac{1}{7}$. Compute $\sin 2x$.
3. $\sin x = \frac{1}{4}$. Compute $\sin 3x$.
4. (FCML) Right triangle ABC (with right angle C) has sidelengths $AB = 10$ and $AC = 6$. An angle bisector of angle A is drawn from A to D on side BC. What is the area of triangle ACD?
5. Compute the exact value of $\cos \frac{16\pi}{13} \cos \frac{8\pi}{13} \cos \frac{4\pi}{13} \sin \frac{9\pi}{13} - \frac{\sin \frac{6\pi}{13}}{8}$.
6. (NYCIML) Compute all θ in degrees such that $0^\circ < \theta < 10^\circ$ and

$$\sin 41\theta = \sin 31\theta + \cos 31\theta + \cos 41\theta.$$

7. $14 \sin x + 84 \cos x = 85$. Let q be the sum of all possible values of $\cos x$, and r be the sum of all possible values of $\sin x$. What is $q - r$?
8. Compute all x in degrees such that $0^\circ < x < 90^\circ$ and $\tan x + \cot x = 4$.
9. $\sin \frac{\alpha}{2} + \cos \frac{\alpha}{2} = \frac{1}{3}$, where $0 \leq \alpha < 4\pi$. Compute all possible values of $\sin \frac{3\alpha}{2} + \cos \frac{3\alpha}{2}$.
10. Let n be a positive integer. Consider $f : \mathbb{N} \rightarrow \mathbb{C}$ that satisfies $f(x) = \prod_{k=1}^x \text{cis}(\frac{k\pi}{n})$, where $\text{cis}(\alpha) = \cos \alpha + i \sin \alpha$, in which $i = \sqrt{-1}$. Obtain a closed for expression equivalent to $\prod_{i=1}^{n-1} f(i)$.
11. Let a_i be a sequence of real numbers defined by $a_n = 2 \sin(\frac{n\pi}{48})a_{n-1}$, and $a_0 = 1$. Compute:

$$\prod_{k=0}^{47} a_k$$

12. Let $p = \prod_{i=1}^{n-1} \cos \frac{i\pi}{n}$, for some odd, positive integer n . Show that $|p| = \frac{1}{2^{n-1}}$.