Applications of Stochastic Processes in Asset Price Modeling

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Abstract & Introduction

Stock market forecasting and asset price modeling have recently become important areas in the financial world today. The increasing complexity of the stock market and the lucrative field of investment management has fueled breakthrough developments in mathematical stock price modeling. One method of mathematical modeling uses random or pseudorandom methods known as stochastic processes to determine an asset's price in the future. This project aims to demonstrate the flexibility and accuracy of these stochastic models by implementing them in code and testing them against empirical data.

Structure

The main model class is responsible for data parsing and simulation. This class reads in historical price data and utilizes a statistical convenience class to calculate inputs to the model. Once these are determined, the simulation process begins. Currently, a Geometric Brownian Motion (GBM) model is being used, but this class can easily be adopted to other models as long as they follow the same convention. This model was implemented using a discrete iterative algorithm to approximate the continuous time forms of the theoretical model. During the simulation, price changes over the given trading period (usually 1 year) are printed out to a file formatted to easily be plotted with Gnuplot. This model supports simultaneous simulations so that several different sample paths for a stock price can be plotted on the same graph.

Results & Conclusions

Looking at the figures to the left, it seems that the GBM stochastic model can readily fit stock price data. In Figure 1, notice how all three simulations nearly match up with the historical price in red. In Figure 2, which took place during a bear session for IBM stock, one can see that the GBM model is capable of simulating negative growth periods even though the rate of drift is positive.

Stochastic Processes

Geometric Brownian Motion SDE:

Stock price returns follow random process

$$\frac{dS}{S} = \mu dt + \sigma dZ$$

$$dZ = \phi\sqrt{dt}$$

Expectation value is based on drift rate, μ

$$\begin{aligned} E(dS) &= E(\sigma S dZ + \mu S dt) \\ &= \mu S dt \\ &\quad \text{since } E(dZ) = 0 \end{aligned}$$

Variance depends on stock volatility, σ

$$\begin{aligned} Var[dS] &= E(dS^2) - [E(dS)]^2 \\ &= E(\sigma^2 S^2 dZ^2) \\ &= \sigma^2 S^2 dt \end{aligned}$$

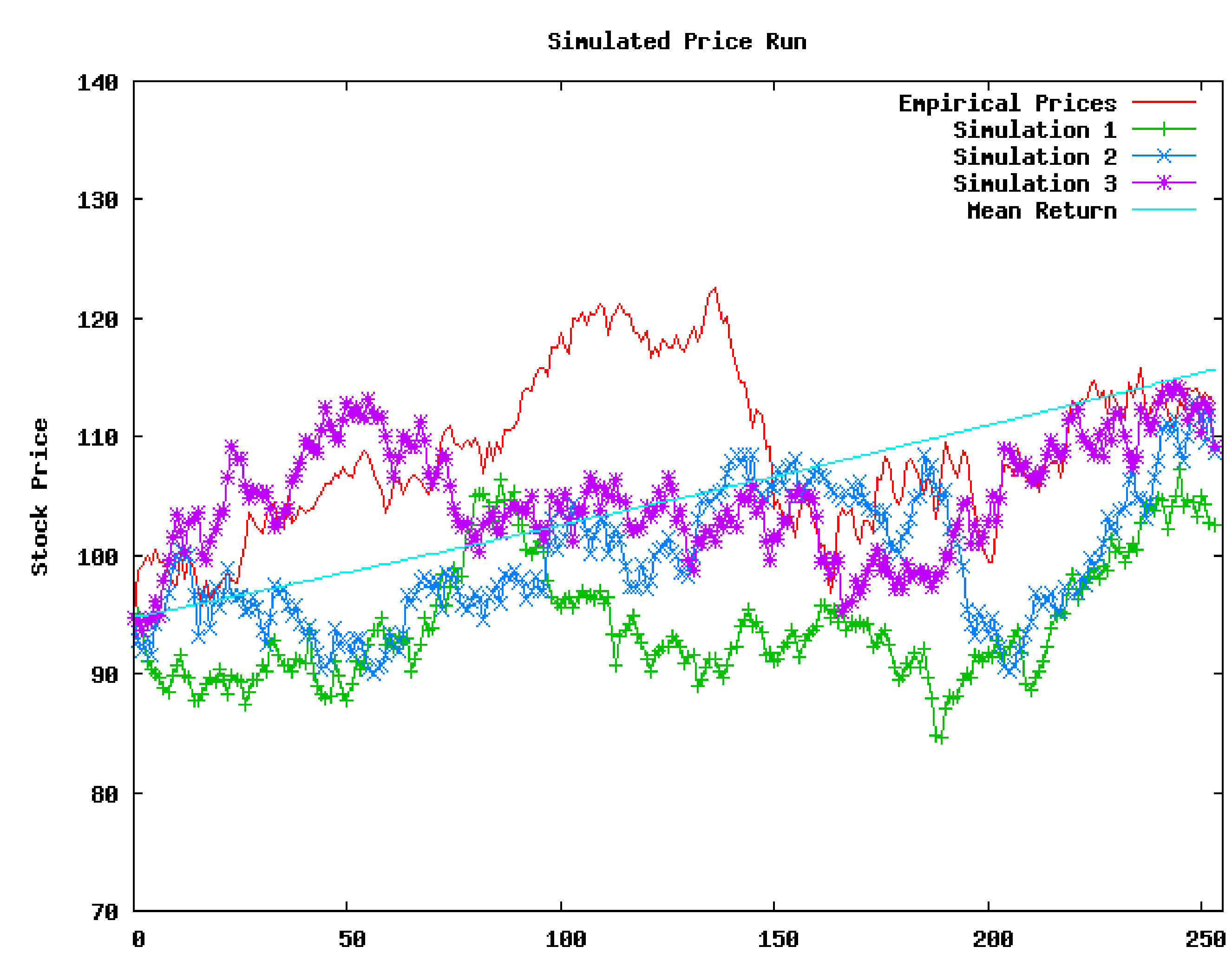

Figure 1: IBM January 1990 - 1991

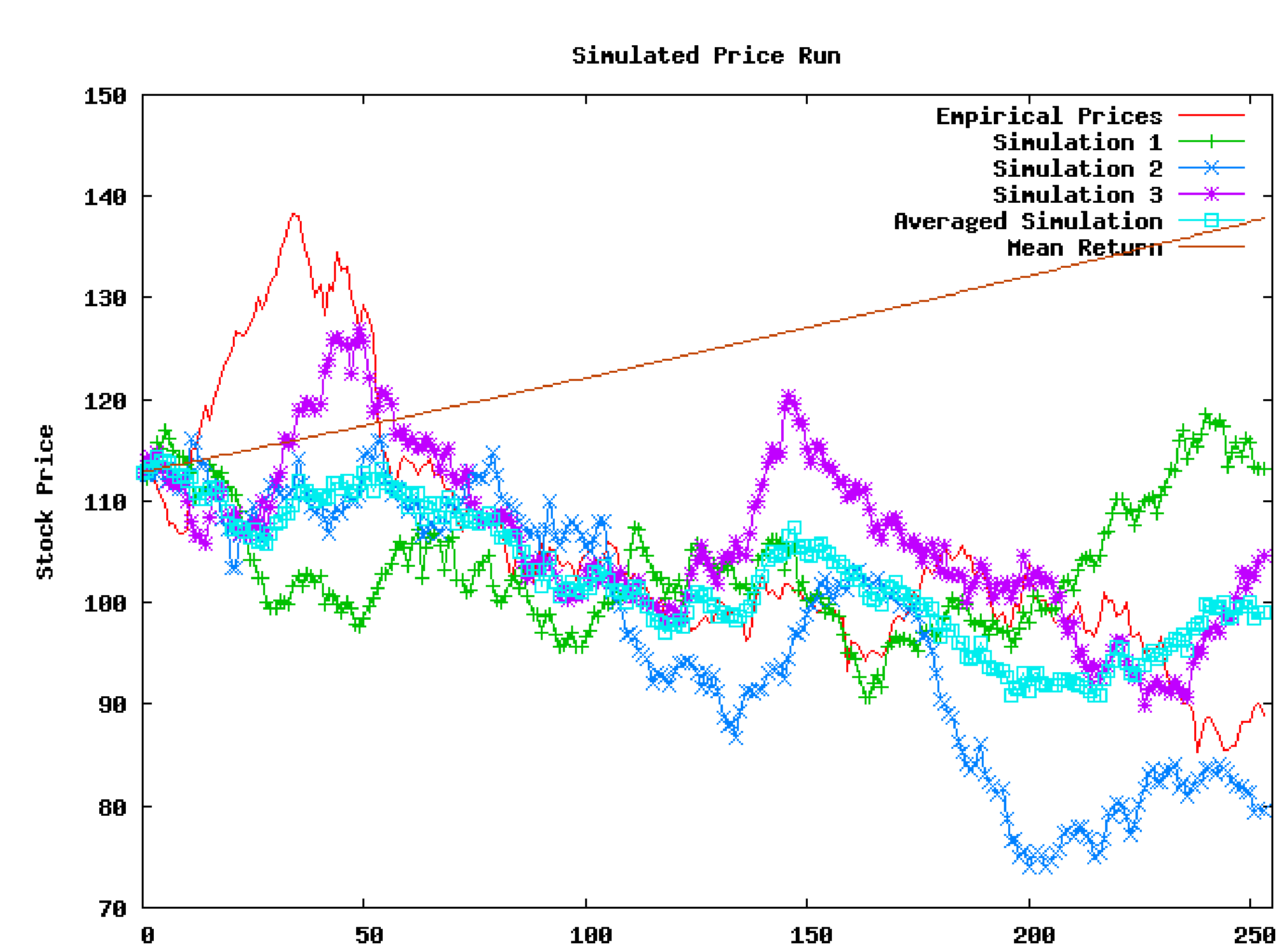

Figure 2: IBM January 1991 - 1992